Journal of the Orissa Mathematical Society ISSN:0975-2323
Volume 33, No.2(2014), p.133-145

Application of fuzzy weighted product method to agricultural allocation problem

A. Baral[1],Sambhunath Behera[2]
[1]Department of Mathematics, Trident Academy of Technology, F2/A,
Chandaka Industrial Estate, Bhubaneswar
Email: a.baral@tat.ac.in
[2]Department of Mathematics, BCET, Sergarh ,Balasore Odisha India τ

Abstract In this paper we have explained some concepts of fuzzy set and applied one fuzzy model on agricultural farm for optimal allocation of different crops by considering maximization of net benefit,Maximization production and Maximization utilization of labour . Crisp values of the objective functions obtained from selected non-dominated solutions are converted into triangular fuzzy numbers and ranking of those fuzzy numbers are done to make a decision.

Keywords and phrases Triangular fuzzy number,α-cut, Optimism index, Weighted Product Method, Multi-criteria Decision Making
2000 Mathematics Subject Classification No 03B52,03E72,28E10

1. Introduction and Literature Survey

In real life situations the introduction of fuzzy logic makes the mathematical models more acceptable for decision makers. We present general discussion on multi criteria decision methods and applied the techniques to one of the important models with their fuzzy extension to the field of Agricultural sciences, where the application of the models seems to be rare. Most of the multi criteria decision methods have not taken full shape or have not been tested.

Mathematical models solved by various methods would provide a comparative analysis of the methods. As for the applicability of the decision analysis by these methods, at the present stage of development, it is probably more useful as means of providing insight rather than analytical answer. It is hoped that the Decision Maker (DM) can make compromises and judgments based on insights generated by multi criteria decision methods. The idea of fuzzy concept was first used in a scientific sense by the computer scientist Lotfi Zadeh in 1965.Fuzzy concept can generate uncertainty because they are imprecise.There are four quite distinct families of method i.e 1- the out ranking, 2-the value and utility theory based, 3-the multiple objective programming and 4-group decision and negotiation theory based method. Fuzzy concept to the extent that their meaning can never be completely and exactly specified with logical operators or objective terms and can have multiple interpretations which are in part exclusively subjective. In this paper, We have discussed the fuzzy Weight Product Method (Fuzzy WPM) as a fuzzy model for decision making in agricultural farm. We have taken an example of a certain agricultural farm in the state of Odisha, India for approximation of fuzzy concept on agricultural land for decision making. The present study deals with the objective of making comparative evaluation of cropping plans so far as allocation of land is concerned. As per Hoda and Kapoor[10]and Chen[4] different area have been selected for different crops in the distribution centre. The application of fuzzy multiobjective linear programming to aggregate production planning has applied by Wang[13]. Here the methodology, so developed,is applied to an existing major irrigation project, Distributary No.1 , Mahanadi-Taladanda Canal, Cuttack, Odisha, India. A total of 18 crops were considered in a pilot area under three conflicting objectives, namely, maximization of net benefit, maximization of agricultural production and maximum utilization of labour.

Different constraints such as land availability, water, fertilizer, labour availability are considered. The response of the farmers and authorities are obtained through a questionnaire. Depending on their response, assessment of weight of each criterion has been obtained. Geometric mean approach is adopted to aggregate the individual opinion to formulate the group opinion. Analytical Hierarchy Process is employed to obtain the weight of the three criteria. Optimization of each individual objective is performed with linear programming algorithm. The pay off matrix is formed to obtain the upper and lower bound of each objective. The maximization of net benefit is taken as the main objective in the constraint method formulation due to the higher importance attributed to it by the farmers and the authorities. Non-dominated solutions are generated by parametrically varying the bounds. Initially, a large number of non-dominated solutions are generated. Different alternatives are ranked and proper weightage are given. Considering the total weights of each alternative few alternatives are selected and cluster analysis is employed to reduce the non-dominated alternatives to a manageable alternatives for more convenient analysis. Then for decision making I have also followed Fullerand and Carlsson[8] principle. The table 2 gives the selected alternative policies for further analysis in MCDM (Multi-Criteria Decision Making) context.

2. Basic Preliminaries

In many decision-making process data play an important role. But in most cases the pertinent data and the sequence of possible actions are not precisely known. Therefore it is required to use fuzzy data to decision-making process. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of crisp numbers. Triangular fuzzy number with lower, modal and upper values has an edge over other fuzzy numbers.

A real fuzzy number M is described as any fuzzy subset of the real line R with membership function μ_m, which possesses the following properties.

(a)μ_m is continuous mapping from R to the closed interval [0, 1]

(b) μ_m (x) = 0 for all x (- , a]

(c) μ_m is strictly increasing on [a, b]

(d)μ_m (x) = 1 for all x ε[b, c]

(e) μ_m is strictly decreasing on [c, d]

(f) μ_m (x) = 0 for all x ε [d,)

where a, b, c, d are real numbers. We may let b = c.

In this work, We have used triangular fuzzy numbers whose membership function

μ_m : R $\rightarrow$ [0, 1] is defined as μ_m(x)=$\frac{x}{m-1}$-$\frac{1}{m-1}$, x ε[1, m]

or μ_m(x)=$\frac{x}{m-u}$-$\frac{u}{m-u}$, x ε[m, u] or μ_m(x)=0 , otherwise

where l$\lneqq$ m $\lneqq$ u and l and u stand for the lower and upper values of the support of the fuzzy number M respectively and m for the modal value. A triangular fuzzy number with lower, modal and upper values is expressed as (l, m, u)

Fuzzy operations were first introduced by Dubois and Prade [6]. Other researchers, such as Laarhoven and Pedrycz [11], Buckley [2] and Boender et al [1]treated a fuzzy version of the AHP by using the fuzzy operations introduced by Dubois and Prade [6].

The basic operations on fuzzy triangular numbers, which were developed and used, are defined as follows.

$\widehat{n}_1 \bigoplus \widehat{n}_2$=($n_{11}$+$n_{21}$,$n_{1m}$+$n_{2m}$,$n_{1u}$+$n_{2u}$) for addition $\widehat{n}_1 \bigoplus \widehat{n}_2$=($n_{11}$×$n_{21}$,$n_{1m}$× n_{2m},$n_{1u} \times n_{2u}$) for multiplication

-$\widehat{n}_1$=$(-n_{1u} - n_{1m}, -n_{11})$ for negation

$\frac{1}{\widehat{n_1}}$=($ln(n_{11})$,$ln(n_{1m})$,$ln(n_{1u})$) for natural logarithm

$exp(\widehat{n_1})$= $exp(\widehat{n_{11}})$,$exp(\widehat{n_{1m}})$,$exp(\widehat{n_{1u}})$ for exponentiation

Where $(\widehat{n_1})=(n_{11},n_{1m},n_{1u})$ and $(\widehat{n_2})=(n_{21},n_{2m},n_{2u})$

represent two triangular fuzzy numbers with lower, modal and upper values and $\cong$ denotes approximation. For the special case of raising of triangular fuzzy number to the power of another triangular fuzzy number, the following approximation was used.

$\widehat{n_1}^{\widehat{n_2}} \cong (\widehat{n_1}^{\widehat{n_2}},\widehat{n_1}^{\widehat{n_2}},\widehat{n_1}^{\widehat{n_2}})$

The problem of ranking fuzzy members appears very often in the literature. As each method of ranking fuzzy numbers has its advantage over the others in certain situations. It is very difficult to determine which method is the best one. Some important factors in deciding which ranking method is the most appropriate for a given situation include the complexity of the algorithm, its flexibility, accuracy, ease of interpretation and the shape of the fuzzy numbers which are used. Baas and Kwakernaak[3] first introduced a method for comparing fuzzy numbers. Detyniecki. and Yager[7] introduced the ranking of fuzzy numbers using alpha weighted valuation .Tong and Boinissone [12] introduced the concept of a dominance measure. Geldermann et al.[9]introduced fuzzy out ranking for environmental assessment. This method was also later adopted by Buckley [2]. According to Zhu and Lee [14] this ranking method is less complex and still effective. It allows a decision maker to implement it without difficulty. However, a given problem may require different method. Here we have discussed ranking of triangular fuzzy numbers using α - cut. In this technique, the irregular fuzzy numbers are further defuzzified into crisp values to determine the order of the alternatives.

Definition of α-cut: The α-cut of fuzzy number M is defined as $M^\alpha = \{x : \mu_m(x) \geq \alpha\}$ where x ϵ R ,α ϵ [0, 1] M^α is a non-empty bounded closed interval contained in R and it can be denoted by $M^\alpha = [M_L^\alpha, M_u^\alpha]$, where M_L^α and M_u^α are the lower and upper bounds of the closed interval respectively.

For example, if M = (a, b, c) be the triangular fuzzy number, then the α cut of M can be expressed as $M^{\alpha} = [M_L^{\alpha}, M_u^{\alpha}]= (b-a)\,\alpha + a, (b-c)\,\alpha + c]$ The graphical representation is shown in Fig 1 Given two fuzzy numbers A and B, A, B ϵR+, the α cuts of A and B are $A^{\alpha} = [A_L^{\alpha}, A_u^{\alpha}]$ $B^{\alpha} = [B_L^{\alpha}, B_u^{\alpha}]$ respectively. Some main operations of A and B can be expressed as follows :
$(A \bigoplus B)^{\alpha} = [A_L^{\alpha} + B_L^{\alpha}, A_U^{\alpha} + B_u^{\alpha}]$
$(A\theta B)^{\alpha} = [A_L^{\alpha} - B_L^{\alpha}, A_U^{\alpha} - B_u^{\alpha}]$
$(A \bigotimes B)^{\alpha} = [A_L^{\alpha}, B_L^{\alpha}, A_U^{\alpha}, B_u^{\alpha}]$
$(A \oslash B)^{\alpha} = [\frac{A_L^{\alpha}}{B_U^{\alpha}}, \frac{A_U^{\alpha}}{B_L^{\alpha}}]$ Chu (2002) introduced a fuzzy number interval arithmetic based fuzzy MCDM algorithm. Using the aboveα cut concept, the fuzzy performance matrices are transformed to interval performance matrices. Theα cut is known to incorporate the experts or decision makers confidence over his preference or the judgment. Theα -cut value ranges from 0 to 1 stating that if the α-cut = 1 then the expert is highly certain about his knowledge regarding a phenomenon over which he expresses his performances and the outcome will be a single value having the membership 1 in the fuzzy performance set. Then the further steps are not needed. But when the αcut is less than 1, it indicates that there exists uncertainty; the expert is obviously uncertain about the decisions he made. The α-cut = 0 expresses the highest levels of uncertainty and then the possible performance will be whole support of the fuzzy performance. Any value of α other than 1 needs further evaluation to get the crisp performance. The crisp performance matrix is obtained by applying the optimism index λ.If represents the interval performance corresponding to a triangular fuzzy number M using αcut, then, the crisp performance c is obtained as $c = \lambda[M_u^{\alpha}] + (1-\lambda)M_L^{\alpha}]$ where $\lambda \in [0, 1]$.

For successful inclusion of uncertainties into the solution procedure, the fuzzy numbers that are used to represent the uncertain model parameters must be implemented in an appropriate form. Considering a definite uncertain parameter a, measured data for the parameter is assumed to be available from which a normalized distribution function can be derived. In most cases, the data approximately show a Gaussian distribution. The uncertainty in the parameter a can be modeled by a fuzzy number $\overline{a}$ with the membership function $\mu_{\overline{a}}$(x) of the form

$\mu_{\overline{a}}(x) = exp(\frac{-(x-m_a)^2}{2\sigma_a^2})$

where m_a and σ_a are the mean value and standard deviation of Gaussian distribution.

The original fuzzy number $\overline{a}$ with the membership function $\mu_{\overline{a}}(x)$ can be approximated by a symmetric triangular fuzzy number $\overline{a}_t$ with the membership function $\mu_{\overline{a_t}}(x)$ that can be obtained by postulating

$\mu_{\overline{a_t}}(m_a)$=$\mu_{\overline{a}}(m_a)$=1

and $\int_{-\infty}^{\infty} \mu_{\overline{a_t}}(x)dx$=$\int_{-\infty}^{\infty} \mu_{\overline{a}}(x)dx$

The membership function $\mu_{\overline{a_t}}$ of the triangular fuzzy number is then defined by $\mu_{\overline{a_t}}(x)$= $max\left\{0, 1-\frac{]x-m_a[}{\sigma}\right\}$

with $\sigma = \sqrt{2\pi}\sigma_a$

which can also be expressed in the form

$\overline{a_t} = m_a - \sigma, m_a, m_a + \sigma$

3. Fuzzy MCDM Method

Initially weight of each criterion is calculated as triangular fuzzy number. Basing on the data collected in form of questionnaire from the farmers and officials, the weights of different criterion is calculated as follows by using the formula

$(mean - \sqrt{2\pi} \times S.D, mean, mean + \sqrt{2\pi} \times S.D)$

Labour: (0.0697, 0.1430, 0.2227)

Production: (0.1897, 0.3260, 0.4623)

Benefit: (0.4249, 0.5310, 0.6371)

where sum of modal values of all criteria is equal to 1and S.D is standard deviation. Then the crisp values of different objectives in decision matrix (Table 3.(a)) are converted into triangular fuzzy numbers.

Fuzzy Weighted Product Method (Fuzzy WPM) In this method, each alternative is compared with others by multiplying a number of ratios, one for each criterion. Each ratio is raised to the power of the relative weight of the corresponding criterion. It is also called dimensionless analysis as it eliminates units of measure. So it can be used in a multi-dimensional decision problem. The best alternative according to this model is the one which satisfies the equation
R $\left(\frac{A_k}{A_1}\right)$ $=\Pi_{j=1}^{N}\left(\frac{A_{kj}}{a_{1j}}\right)w_j$' where $\widehat{a}_{kj}$, $\widehat{a}_{lj}$ and $\widehat{a}_{kj}$ are triangular fuzzy numbers. Alternative A_k dominates alternative A_1 if and only if the numerator in the above equation is greater than the denominator.

Considering the decision matrix given in the Table 2, the calculation are made as follows. R $\left(\frac{RP1}{RP2}\right) = \frac{N_1}{N_2} = \frac{2.098312, 2.91445, 4.191345}{2.115837, 2.94623, 4.24684}$
where, $N1 = [(1.018337, 1.127460, 1.236583)^{(0.0697, 0.1430, 0.2227)}$
$\times(2.73395, 2.879124, 3.024298)^{(0.1897, 0.326, 0.4623)}$
$\times(3.640964, 3.792134, 3.943304)^{(0.4249, 0.531, 0.6371)}]$
and $N2 = [(1.012957, 1.122080, 1.231203)^{(0.0697, 0.1430, 0.2227)}$
$\times$ $(2.770999, 2.916173, 3.061347)^{(0.1897, 0.326, 0.4623)}$
$\times(3.693898, 3.845068, 3.996238)^{(0.4249, 0.531, 0.6371)}]$ Similarly other ratios are obtained and the fuzzy numbers NI are given in the table 3.

Then, interval performance matrix by using α-cut over the fuzzy numbers N1, N2, . . . ,N6 is obtained by taking the confidence limit 60 percentage. i.e.

$\alpha = 0.6$ and presented in Table 3. Then crisp performance matrix is obtained by applying the optimism index λ for three different values as 0.3, 0.5, and 0.7 and presented in Table 4. The ranks of different policies for different values of λ are calculated using the crisp performance matrix and presented in Table 5. It is observed from the above table that ranking pattern for the policies is same for three different values of λ and the policies RP6 and RP5 occupy the first and second rank respectively.

4. **Conclusion**

In this paper, we have applied one fuzzy decision making process to an agricultural farm for allocation of land for 18 crops to get maximum net benefit, maximum agricultural production and maximum utilization of agricultural labour. On few chosen policies, the fuzzy MCDM method is applied and it is found that one particular policy bags the first rank, which can be taken as the best compromising solution.

Table 1. Pay-off Matrix

Criteria Policies	Labour in lakhs man days	Production in lakhs of quintals	Net Benefit in crores of rupees
P1	1.127460	2.8791235	3.7921340
P2	1.122080	2.9161725	3.8450680
P3	1.140924	2.9717465	3.6726790
P4	1.109613	2.9902711	3.6507440
P5	1.178825	2.7827819	4.0853980
P6	1.150925	2.8229123	4.0931130

Table 2. Fuzzy decision matrix.

Policies	**Labour**			**Production**			**Benefit**		
	Lower	**Modal**	**Upper**	**Lower**	**Modal**	**Upper**	**Lower**	**Modal**	**Upper**
RP1	1.018337	1.127460	1.236583	2.733950	2.879124	3.024298	3.640964	3.792134	3.943304
RP2	1.012957	1.122080	1.231203	2.770999	2.916173	3.061347	3.693898	3.845068	3.996238
RP3	1.065521	1.140924	1.216327	2.889852	2.971747	3.053642	3.511859	3.672679	3.833499
RP4	1.034210	1.109613	1.185016	2.908376	2.990271	3.072166	3.489924	3.650744	3.811564
RP5	1.083944	1.178825	1.273706	2.640862	2.782782	2.924702	3.983247	4.085398	4.187549
RP6	1.056044	1.150925	1.245806	2.680992	2.822912	2.964832	3.990962	4.093113	4.253933

Table 3.

N1	(2.098312 ,2.914450 ,4.191345)
N2	(2.115837 ,2.946230 ,4.246840)
N3	(2.094826,2.899983,4.119844)
N4	(2.087446,2.885132,4.092462)
N5	(2.175134,3.017748,4.316374)
N6	(2.179195,3.024528,4.365811)

Table 4. Interval performance matrix

Policies	Lower bound	Upper bound
RP1	2.587995	3.425208
RP2	2.614072	3.466473
RP3	2.644496	3.516087
RP4	2.566058	3.368065
RP5	2.680702	3.537198
RP6	2.686395	3.561041

Table 5. Crisp performance matrix

Policies	**λ=0.3**	**λ =0.5**	**λ =0.7**
RP1	2.839158	3.006601	3.174044
RP2	2.869793	3.040273	3.210753
RP3	2.905973	3.080291	3.254610
RP4	2.806660	2.967061	3.127463
RP5	2.937651	3.108950	3.280249
RP6	2.948789	3.123718	3.298647

Table 6. Final rank of policies by fuzzy WPM method

Policy	**Rank for λ = 0.3**	**Rank for λ = 0.5**	**Rank for λ = 0.7**
RP1	5	5	5
RP2	4	4	4
RP3	3	3	3
RP4	6	6	6
RP5	2	2	2
RP6	1	1	1

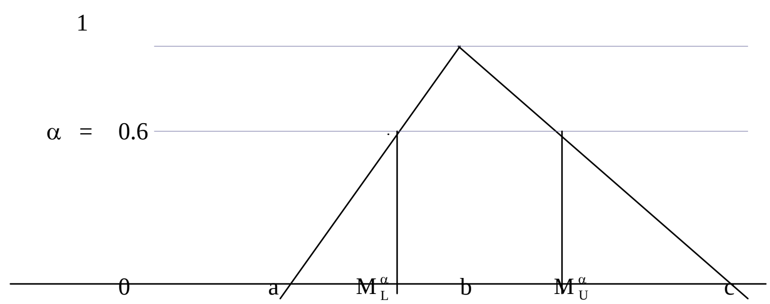

Fig 1. Alpha cut operation on triangular fuzzy number M = (a, b, c)

References

[1] Boender, C. G. E. Graan, J. G. de and Lootsman, F. A., A multi-criteria decision analysis with fuzzy pair wise comparisons. Fuzzy sets and Systems, 29, 133-143, 1989.

[2] Buckley, J.J., Ranking alternatives using fuzzy numbers. Fuzzy Sets and Systems, 15, 21-31, 1985.

[3] Bass, S. J. and Kwakernaak, H., Rating and ranking of multi-aspect alternatives using fuzzy sets. Automatica, 13, 47-58, 1977.

[4] Chen, C.T., A fuzzy approach to select the location of the distribution centre. Fuzzy sets and systems, 118, 65-73, 2001.

[5] Chu, Ta-Chung, A fuzzy number Interval Arithmetic based fuzzy MCDM Algorithm. International Journal of Fuzzy System, 4, 867-872, 2002.

[6] Dubois, D. and Prade, H., *Fuzzy sets and systems, Theory and Application*, Academic Press, New York, 1980.

[7] Detyniecki, M. and Yager, R.R., Ranking fuzzy numbers using alpha weighted valuation. International Journal of Uncertainty, Fuzziness and Knowledge-Based System, 8, 573-592, 2001.

[8] Fuller, R. and Carlsson, C. Fuzzy multiple criteria decision making recent development, Fuzzy sets and system, 78, 139-153, 1996.

[9] Geldermann, J., Spengler, T. and Otto, R., Fuzzy outranking for environmental assessment. Case study : iron and steel making industry, Fuzzy sets and system, 115, 45-65, 2000.

[10] Hoda, D.S. and Kapoor, J.N., Crop Area Distributions for optimal yield OPSEARCH, 38, 407-418, 2001.

[11] Laarhoven, P. J. M. and Pedrycz, W., A fuzzy extension of Sattys priority theory, Fyzzy Sets and Systems, 11, 229-241, 2001.

[12] Tong, R. M. and Bonissone, P. P., A linguistic approach to decision making with fuzzy sets, IEEE Trans. Systems man cybernet, SMC. 10, 716-723, 1980.

[13] Wang, Reay-chan and Liang, Tien-fu, Application of fuzzy multi-objective linear programming to aggregate production planning, Computers and Industrial Engineering, 46, 17-41, 2004.

[14] Zhu, Q. and Lee, E. S., Comparison and ranking of fuzzy numbers, Fuzzy Regression Analysis, Omnitech Press, Warsaw., 132-145, 1992.