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Unsteady MHD Free Convection Flow of a Heat Radiating Fluid past a Flat Plate With Rampedness in Wall Temperature and Species Concentration

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Abstract The problem of unsteady free convection flow and mass transfer of a viscous, incompressible, electrically conducting and heat radiating fluid under the influence of a uniform transverse magnetic field is studied. Fluid flow is induced due to a general time dependent motion of the infinite vertical flat plate which is having a temporarily ramped temperature profile. The species concentration is also assumed to have temporary rampedness. The exact solution of the governing equations are obtained analytically using Laplace transform technique. To highlight the effects of rampedness in wall temperature and species concentration, exact solution is also obtained, considering the wall temperature and species concentration as constant. Some important applications of practical interest are discussed for different types of motion of the plate . The numerical values of species concentration, temperature and velocity of the fluid are displayed graphically for various values of pertinent flow parameters whereas the effects of different flow parameters on the rate of mass transfer at the plate, rate of heat transfer and the skin friction are presented in tabular form.

Keywords and phrases Free convection, mass transfer, magnetic field, radiation, ramped temperature, ramped species concentration.

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1. Introduction

The simultaneous effects of convective heat and mass transfer on the flow of a viscous, incompressible and electrically conducting fluid in the presence of a magnetic field with or without heat absorption/generation or radiation has many engineering and geophysical applications such as geothermal reservoir, enhanced oil recovery, drying of porous solids, thermal insulation, underground energy transport and cooling of nuclear reactor. Considering these applications, MHD free convection flow with mass transfer effect has been investigated extensively by many researchers. Hossain and Mandal [1] investigated the effects of mass transfer on the unsteady MHD free convection flow past an accelerated vertical porous plate. Jha [2] studied the free convection MHD flow with heat transfer effects through a porous medium. Elbashbeshy [3] did the research on the heat and mass transfer along a vertical flat plate in the presence of magnetic field with variable surface tension and concentration. Chamkha and Khaled [4] investigated the combined effects of heat and mass transfer by natural convection of a hydromagnetic fluid from a permeable surface embedded in a saturated porous medium. Chamkha and Khaled [5] presented the similarity solution for hydromagnetic heat and mass transfer by natural convection from an inclined plate with heat generation or absorption. Chen [6] studied the combined effect of heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation. Afify [7] discussed the MHD free convective heat and mass transfer flow over a stretching sheet in the presence of suction/injection with thermal diffusion and diffusion thermo effects. Eldabe et al. [8] studied the unsteady flow of hydromagnetic viscous incompressible fluid with heat and mass transfer through porous medium near a moving vertical plate. Kamel [9] studied the effects of heat source/sink on the unsteady MHD convection through porous medium with combined heat and mass transfer. Chamkha [10] solved the problem of unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving flat plate with heat absorption. Mbeledogu and Ogulu [11] studied the heat and mass transfer effects on unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. Makinde [12] discussed the hydromagnetic boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. Pal and Mondal [13] studied the effects of Soret Dufour, thermal radiation and chemical reaction on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet.

In the investigations made above, the numerical or analytical solution is obtained assuming conditions for velocity and temperature at the interface of the plate as well defined and continuous. However, there exist several problems of physical interest which may require non-uniform or arbitrary wall conditions. Taking into account these facts, several researchers [14, 15, 16, 17] investigated the problems of free convection from a vertical plate with step-discontinuities in the surface temperature. Chandran et al. [18] considered unsteady natural convection flow of a viscous incompressible electrically conducting fluid past a vertical plate with ramped wall temperature. Seth et al. [19] studied unsteady natural convection flow of a viscous incompressible electrically conducting fluid near an impulsively moving vertical plate in a porous medium with ramped wall temperature taking into account the effects of thermal radiation. They compared the results of natural convection near a ramped temperature plate with that of natural convection near an isothermal plate. Seth et al. [20] extended this problem to include the effects of rotation. Recently, Nandkeolyar et al. [21] studied the solution of unsteady hydromagnetic free convection in a heat absorbing fluid flow past a flat plate with rampedness in wall temperature. Again in the same year Nandkeolyar et al. [22] did the study on unsteady MHD flow of a heat radiating and chemically reactive fluid past a flat porous plate with ramped wall temperature with heat and mass transfer effects. They did the investigations on both suction and blowing.

The aim of the present paper is to study the hydromagnetic free convective heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid past a vertical infinite flat plate with ramped wall temperature in the presence of ramped species concentration. The fluid flow is assumed to be induced due to a general time dependent movement of the infinite flat plate. The governing equations, describing the model, are solved analytically and a general solution has been obtained which is valid for any time dependent movement of the plate. Some particular cases of the general solution are discussed with a view to highlight the applications of the present paper.

2. Formulation of the Problem

Consider the unsteady free convection heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid along an infinite non-conducting vertical flat plate in the presence of a uniform magnetic field B_0 applied in a transverse direction to the fluid flow. Let x-axis be along the plate in upward direction, y-axis is normal to it and $z-$axis is normal to $xy-$plane.

Initially, for time $t' \leq 0$, the plate and the fluid are at the same constant temperature T'_∞ in a stationary condition with the same species concentration C'_∞. Subsequently, for $t' > 0$ the plate is assumed to be accelerated with a time dependent velocity $U_0 f(t')$ in its own plane along the $x-$axis, instantaneously the temperature of the plate and the species concentration is raised or lowered to $T'_\infty + (T'_w - T'_\infty)t'/t_0$ and $C'_\infty + (C'_w - C'_\infty)t'/t_0$, respectively when $t' < t_0$ and thereafter, for $t' > t_0$, is maintained at uniform temperature T'_w and uniform species concentration C'_w. Since the plate is of infinite extent in x and z directions and is electrically nonconducting, all the physical quantities except pressure are function of y and t only. It is further assumed that there is no slip occurs between the plate and fluid.

Under the above assumptions, the governing equations for the fully developed hydromagnetic free convection heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid are given by

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u' + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty), \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q'_r}{\partial y}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2}, \tag{3}$$

where u', ρ, g, β, β', T', C', c_p, k, q_r, ν, σ and D are, respectively, fluid velocity in x direction, fluid density, acceleration due to gravity, volumetric co-efficient of thermal expansion, volumetric coefficient of expansion or contraction, temperature of the fluid near the plate, species concentration, specific heat at constant pressure, thermal conductivity, radiative flux, kinematic co-efficient of viscosity, electrical conductivity and chemical molecular diffusivity.

The corresponding initial and boundary conditions are

$$u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for } y \geq 0 \text{ and } t' \leq 0, \tag{4a}$$

$$u' = U_0 f(t') \text{ at } y = 0 \text{ for } t' > 0, \tag{4b}$$

$$T' = T'_\infty + (T'_w - T'_\infty)t'/t_0 \text{ at } y = 0 \text{ for } 0 < t' \leq t_0, \tag{4c}$$

$$T' = T'_w \text{ at } y = 0 \text{ for } t' > t_0, \tag{4d}$$

$$C' = C'_\infty + (C'_w - C'_\infty)t'/t_0 \text{ at } y = 0 \text{ for } 0 < t' \leq t_0, \tag{4e}$$

$$C' = C'_w \text{ at } y = 0 \text{ for } t' > t_0, \tag{4f}$$

$$u' \to 0, T' \to T'_\infty, C' \to C'_\infty \text{ as } y \to \infty \text{ for } t' > 0. \tag{4g}$$

For an optically thick fluid, in addition to emission there is also self-absorption and usually the absorption coefficient is wavelength dependent and large so that we can adopt Rosseland approximation for radiative flux vector q'_r . The radiative flux vector q'_r under Rosseland approximation becomes

$$q'_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y}, \tag{5}$$

where k^* is mean absorption coefficient and σ^* is Stefan Boltzmann constant. Assuming small temperature difference between fluid temperature T' and free stream temperature T'_∞, T'^4 is expanded in Taylor series about a free stream temperature T'_∞. Neglecting second and higher order terms in $(T' - T'_\infty)$, we obtain

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{6}$$

Making use of Eqs. 5 and 6, in Eq. 2, we obtain

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y^2} + \frac{1}{\rho c_p}\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y^2}. \tag{7}$$

In order to reduce the governing equations 1, 3 and 7, into non-dimensional form, the following dimensionless variables and parameters are introduced

$$\left.\begin{aligned} \eta =& y/U_0 t_0, u = u'/U_0, t = t'/t_0, T = (T' - T'_\infty)/(T'_w - T'_\infty),\\ C =&(C' - C'_\infty)/(C'_w - C'_\infty), M = \sigma B_0^2 \nu/\rho U_0^2, Gr = g\beta^*\nu(T'_w - T'_\infty)/U_0^3,\\ Gm =& g\beta'\nu(C'_w - C'_\infty)/U_0^3, Pr = \rho\nu c_p/k, Sc = \frac{\nu}{D}, \text{ and } N = 16\sigma^* T'^3_\infty/3kk^*. \end{aligned}\right\} \tag{8}$$

The governing equations 1, 3 and 7 in non-dimensional form, become

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \eta^2} + G_r T + G_m C - Mu, \tag{9}$$

$$\frac{\partial T}{\partial t} = \frac{(1+N)}{Pr}\frac{\partial^2 T}{\partial \eta^2}, \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial \eta^2}, \tag{11}$$

where, M is the magnetic parameter, Gr is the thermal Grashof number, Gm is the mass Grashof number, Pr is the Prandtl number, Sc is the Schmidt number and N is the radiation parameter.

According to the above non-dimensionalization process, the characteristic time t_0 can be defined as,

$$t_0 = \frac{\nu}{U_0^2}. \tag{12}$$

The corresponding initial and boundary conditions 4, in non-dimensional form become

$$u = 0, T = 0, C = 0 \text{ for } \eta \geq 0 \text{ and } t \leq 0, \tag{13a}$$

$$u = f(t) \text{ at } \eta = 0 \text{ for } t > 0, \tag{13b}$$

$$T = t \text{ at } \eta = 0 \text{ for } 0 < t \leq 1, \tag{13c}$$

$$T = 1 \text{ at } \eta = 0 \text{ for } t > 1, \tag{13d}$$

$$C = t \text{ at } \eta = 0 \text{ for } 0 < t \leq 1, \tag{13e}$$

$$C = 1 \text{ at } \eta = 0 \text{ for } t > 1, \tag{13f}$$

$$u \to 0, T \to 0, C \to 0 \text{ as } \eta \to \infty \text{ for } t > 0 \tag{13g}$$

The system of equations 9-11 of differential equations, subject to the initial and boundary conditions 13, describes the model of free convection heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid near a moving vertical plate with ramped wall temperature and ramped species concentration.

3. Solution of the problem

The system of equations 9-11 subject to the initial and boundary conditions prescribed in 13 are solved analytically using Laplace transform technique and the expressions for species concentration $C(\eta, t)$, fluid temperature $T(\eta, t)$ and fluid velocity $u(\eta, t)$, respectively, are presented as

$$C(\eta, t) = P_1(\eta, t) - H(t-1)P_1(\eta, t-1), \tag{14}$$

$$T(\eta, t) = P_2(\eta, t) - H(t-1)P_2(\eta, t-1), \tag{15}$$

$$\begin{aligned} u(\eta, t) =& P(\eta, t) + \alpha_1 \left[P_3(\eta, t) - H(t-1)P_3(\eta, t-1)\right] + \alpha_2 \left[P_4(\eta, t)\right] \\ &- \alpha_2 \left[H(t-1)P_4(\eta, t-1)\right], \end{aligned} \tag{16}$$

where

$$P_1(\eta,t) = \left(t + \frac{a\eta^2}{2}\right)\ \mathrm{erfc}(t_1) - \sqrt{\frac{at}{\pi}}\ \eta e^{-\frac{a\eta^2}{4t}}, \tag{17a}$$

$$P_2(\eta,t) = \left(t + \frac{b\eta^2}{2}\right)\ \mathrm{erfc}(t_2) - \sqrt{\frac{bt}{\pi}}\ \eta e^{-\frac{b\eta^2}{4t}}, \tag{17b}$$

$$\begin{aligned} P_3(\eta,t) = & \left(\frac{\alpha t - 1}{\alpha^2} + \frac{a\eta^2}{2\alpha}\right)\ \mathrm{erfc}(t_1) - \frac{1}{\alpha}\sqrt{\frac{at}{\pi}}\eta e^{-\frac{a\eta^2}{4t}} \\ & + \frac{e^{-\alpha t}}{2\alpha^2}\left[e^{\eta\sqrt{-a\alpha}}\ \mathrm{erfc}(t_3) + e^{-\eta\sqrt{-a\alpha}}\ \mathrm{erfc}(t_4)\right] \\ & - \left[\left(\frac{\alpha t - 1}{2\alpha^2} + \frac{\eta}{4\alpha\sqrt{M}}\right) e^{\eta\sqrt{M}}\ \mathrm{erfc}(t_5)\right. \\ & + \left(\frac{\alpha t - 1}{2\alpha^2} - \frac{\eta}{4\alpha\sqrt{M}}\right) e^{-\eta\sqrt{M}}\ \mathrm{erfc}(t_6) \\ & - \frac{e^{-\alpha t}}{2\alpha^2}\left[e^{\eta\sqrt{M-\alpha}}\ \mathrm{erfc}(t_7) + e^{-\eta\sqrt{M-\alpha}}\ \mathrm{erfc}(t_8)\right], \end{aligned} \tag{17c}$$

$$\begin{aligned} P_4(\eta,t) = & \left(\frac{\xi t - 1}{\xi^2} + \frac{b\eta^2}{2\xi}\right)\ \mathrm{erfc}(t_2) - \frac{1}{\xi}\sqrt{\frac{bt}{\pi}}\eta e^{-\frac{b\eta^2}{4t}} \\ & + \frac{e^{-\xi t}}{2\xi^2}\left[e^{\eta\sqrt{-b\xi}}\ \mathrm{erfc}(t_9) + e^{-\eta\sqrt{-b\xi}}\ \mathrm{erfc}(t_{10})\right] \\ & - \left[\left(\frac{\xi t - 1}{2\xi^2} + \frac{\eta}{4\xi\sqrt{M}}\right) e^{\eta\sqrt{M}}\ \mathrm{erfc}(t_5)\right] \\ & + \left(\frac{\xi t - 1}{2\xi^2} - \frac{\eta}{4\xi\sqrt{M}}\right) e^{-\eta\sqrt{M}}\ \mathrm{erfc}(t_6) \\ & - \frac{e^{-\xi t}}{2\xi^2}\left[e^{\eta\sqrt{M-\xi}}\ \mathrm{erfc}(t_{11}) + e^{-\eta\sqrt{M-\xi}}\ \mathrm{erfc}(t_{12})\right], \end{aligned} \tag{17d}$$

$$t_1 = \frac{\eta}{2}\sqrt{\frac{a}{t}}; t_2 = \frac{\eta}{2}\sqrt{\frac{b}{t}}; t_3, t_4 = \pm\sqrt{-\alpha t} + \frac{\eta}{2}\sqrt{\frac{a}{t}}; t_5, t_6 = \pm\sqrt{Mt} + \frac{\eta}{2}\frac{1}{\sqrt{t}}$$

$$t_7, t_8 = \pm\sqrt{(M-\alpha)t} + \frac{\eta}{2}\frac{1}{\sqrt{t}}; t_9, t_{10} = \pm\sqrt{-\xi t} + \frac{\eta}{2}\sqrt{\frac{b}{t}};$$

$$t_{11}, t_{12} = \pm\sqrt{(M-\xi)t} + \frac{\eta}{2}\frac{1}{\sqrt{t}}, \tag{18a}$$

$$a = S_c, b = \frac{P_r}{1+N}, \alpha = \frac{M}{1-a}, \xi = \frac{M}{1-b}, \alpha_1 = \frac{G_m}{1-a} \text{ and } \alpha_2 = \frac{G_r}{1-b}. \tag{18b}$$

4. Solution of the problem with isothermal plate and uniform species concentration

Equations 14-18 represent the analytical solutions for the hydromagnetic free convective heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid in the presence of ramped wall temperature and concentration. In order to highlight the effects of rampedness in wall temperature of the plate and species concentration on the fluid flow, it is worth while to compare such a flow with near a moving plate with constant temperature and constant species concentration. Taking into considerations made in section 2, the solution for the species concentration $C(\eta, t)$, fluid temperature $T(\eta, t)$ and the fluid velocity $u(\eta, t)$ for natural convection flow near an isothermal plate in the presence of uniform species concentration is presented in the following form

$$C(\eta, t) = \operatorname{erfc}(t_1), \tag{19}$$

$$T(\eta, t) = \operatorname{erfc}(t_2), \tag{20}$$

$$u(\eta, t) = P(\eta, t) + \alpha_1 P_5(\eta, t) + \alpha_2 P_6(\eta, t), \tag{21}$$

where,

$$\begin{aligned}P_5(\eta,t) =& \frac{1}{\alpha}\ \mathrm{erfc}(t_1) - \frac{e^{-\alpha t}}{2\alpha}\left(e^{\eta\sqrt{-a\alpha}}\ \mathrm{erfc}(t_3) + e^{-\eta\sqrt{-a\alpha}}\ \mathrm{erfc}(t_4)\right)\\ &- \frac{1}{2\alpha}\left(e^{\eta\sqrt{M}}\ \mathrm{erfc}(t_5) + e^{-\eta\sqrt{M}}\ \mathrm{erfc}(t_6)\right)\\ &+ \frac{e^{-\alpha t}}{2\alpha}\left(e^{\eta\sqrt{M-\alpha}}\ \mathrm{erfc}(t_7) + e^{-\eta\sqrt{M-\alpha}}\ \mathrm{erfc}(t_8)\right), \qquad (22a)\end{aligned}$$

$$\begin{aligned}\text{and } P_6(\eta,t) =& \frac{1}{\xi}\ \mathrm{erfc}(t_2) - \frac{e^{-\xi t}}{2\xi}\left(e^{\eta\sqrt{-b\xi}}\ \mathrm{erfc}(t_9) + e^{-\eta\sqrt{-b\xi}}\ \mathrm{erfc}(t_{10})\right)\\ &- \frac{1}{2\xi}\left(e^{\eta\sqrt{M}}\ \mathrm{erfc}(t_5) + e^{-\eta\sqrt{M}}\ \mathrm{erfc}(t_6)\right)\\ &+ \frac{e^{-\xi t}}{2\xi}\left(e^{\eta\sqrt{M-\xi}}\ \mathrm{erfc}(t_{11}) + e^{-\eta\sqrt{M-\xi}}\ \mathrm{erfc}(t_{12})\right). \qquad (22b)\end{aligned}$$

5. Applications of the Problem

The equations 14-22 represent the analytical solutions for the flow induced due to a general time dependent movement of the plate. In order to gain some practical insight into the flow problems some particular cases are discussed below.

5.1. **Movement of the plate with uniform velocity.** Assuming that the plate moves with uniform velocity $f(t) = H(t)$ the fluid velocity for ramped temperature plate in the presence of ramped species concentration is obtained as

$$\begin{aligned}u(\eta,t) =& P_7(\eta,t) + \alpha_1\left[P_3(\eta,t) - H(t-1)P_3(\eta,t-1)\right] + \alpha_2\left[P_4(\eta,t)\right.\\ &\left. - H(t-1)P_4(\eta,t-1)\right), \qquad (23)\end{aligned}$$

and the fluid velocity for isothermal plate in the presence of uniform species concentration is presented as

$$u(\eta,t) = P_7(\eta,t) + \alpha_1 P_5(\eta,t) + \alpha_2 P_6(\eta,t), \qquad (24)$$

where,

$$P_7(\eta, t) = \frac{1}{2}\{e^{\eta\sqrt{M}} \operatorname{erfc}(t_5) + e^{-\eta\sqrt{M}} \operatorname{erfc}(t_6)\}. \tag{25}$$

5.2. **Movement of the plate with single acceleration.** Assuming that the plate moves with single acceleration $f(t) = tH(t)$ the fluid velocity for ramped temperature plate in the presence of ramped species concentration is obtained as

$$\begin{aligned} u(\eta, t) =& P_8(\eta, t) + \alpha_1 \left[P_3(\eta, t) - H(t-1)P_3(\eta, t-1)\right] + \alpha_2 \left[P_4(\eta, t)\right. \\ & \left. - H(t-1)P_4(\eta, t-1)\right], \end{aligned} \tag{26}$$

and the fluid velocity for isothermal plate in the presence of uniform species concentration is presented as

$$u(\eta, t) = P_8(\eta, t) + \alpha_1 P_5(\eta, t) + \alpha_2 P_6(\eta, t), \tag{27}$$

where,

$$P_8(\eta, t) = \left(\frac{t}{2} + \frac{\eta}{4\sqrt{M}}\right) e^{\eta\sqrt{M}} \operatorname{erfc}(t_5) + \left(\frac{t}{2} - \frac{\eta}{4\sqrt{M}}\right) e^{-\eta\sqrt{M}} \operatorname{erfc}(t_6). \tag{28}$$

5.3. **Oscillatory movement of the plate.** Assuming that the plate moves with periodic acceleration $f(t) = \cos\omega t H(t)$ the fluid velocity for ramped temperature plate in the presence of ramped species concentration is obtained as

$$\begin{aligned} u(\eta, t) =& P_9(\eta, t) + \alpha_1 \left[P_3(\eta, t) - H(t-1)P_3(\eta, t-1)\right] + \alpha_2 \left[P_4(\eta, t)\right. \\ & \left. - H(t-1)P_4(\eta, t-1)\right], \end{aligned} \tag{29}$$

and the fluid velocity for isothermal plate in the presence of uniform species concentration is presented as

$$u(\eta, t) = P_9(\eta, t) + \alpha_1 P_5(\eta, t) + \alpha_2 P_6(\eta, t), \tag{30}$$

where,

$$\left.\begin{aligned} P_9((\eta,t) =&\frac{1}{4}e^{-i\omega t}\left[e^{\eta\sqrt{M-i\omega}}\ \mathrm{erfc}\left(\sqrt{(M-i\omega)t}+\frac{\eta}{2\sqrt{t}}\right)\right]\\ &+\frac{1}{4}e^{-i\omega t}\left[e^{-\eta\sqrt{M-i\omega}}\ \mathrm{erfc}\left(-\sqrt{(M-i\omega)t}+\frac{\eta}{2\sqrt{t}}\right)\right]\\ &+\frac{1}{4}e^{i\omega t}\left[e^{\eta\sqrt{M+i\omega}}\ \mathrm{erfc}\left(\sqrt{(M+i\omega)t}+\frac{\eta}{2\sqrt{t}}\right)\right]\\ &+\frac{1}{4}e^{i\omega t}\left[e^{-\eta\sqrt{M+i\omega}}\ \mathrm{erfc}\left(-\sqrt{(M+i\omega)t}+\frac{\eta}{2\sqrt{t}}\right)\right].\end{aligned}\right] \tag{31}$$

6. Sherwood number, Nusselt number and Skin friction

6.1. Sherwood number. Sherwood number which measures the rate of mass transfer at the plate, in the presence of ramped species concentration, is given by

$$Sh = -\left(\frac{\partial C}{\partial \eta}\right)_{\eta=0} = -\{F_1(t) - H(t-1)F_1(t-1)\}, \tag{32}$$

and in the presence of uniform species concentration is presented as

$$Sh_i = \sqrt{\frac{a}{\pi t}}, \tag{33}$$

where,

$$F_1(t) = -2\sqrt{\frac{at}{\pi}}. \tag{34}$$

6.2. Nusselt number. Nusselt number which is a measure of rate of heat transfer at the plate is presented for ramped temperature plate in the presence of ramped species concentration as

$$Nu = -\left(\frac{\partial T}{\partial \eta}\right)_{\eta=0} = -\left[F_2(t) - H(t-1)F_2(t-1)\right], \tag{35}$$

and that for isothermal plate in the presence of uniform species concentration, is obtained as

$$Nu_i = \sqrt{\frac{b}{\pi t}}, \tag{36}$$

where,

$$F_2(t) = -2\sqrt{\frac{bt}{\pi}}. \tag{37}$$

6.3. **Skin friction.** Skin friction which measures the shear stress at the plate is discussed below for different types of time dependent movement of the plate.

6.3.1. *Movement of the plate with uniform velocity.* When the plate moves with uniform velocity, the skin friction for ramped temperature plate in the presence of ramped species concentration is expressed as

$$\begin{aligned}\tau_1 = -\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} &= -Q_1(t) - \alpha_1\{F_3(t) - H(t-1)F_3(t-1)\} \\ &- \alpha_2\{F_4(t) - H(t-1)F_4(t-1)\},\end{aligned} \tag{38}$$

and the skin friction for isothermal plate in the presence of uniform species concentration is obtained as

$$\tau_{1i} = -Q_1(t) - \alpha_1 F_5(t) - \alpha_2 F_6(t), \tag{39}$$

where,

$$Q_1(t) = -\sqrt{M}\ \mathrm{erf}(\sqrt{Mt}) - \frac{e^{-Mt}}{\sqrt{\pi t}}, \tag{40a}$$

$$F_3(t) = -\frac{\sqrt{-a\alpha}}{\alpha^2}\ e^{-\alpha t}\ \mathrm{erf}(\sqrt{-\alpha t}) - \frac{2}{\alpha}\sqrt{\frac{at}{\pi}} \tag{40b}$$

$$+\left(\frac{\alpha t-1}{\alpha^2}\sqrt{M} + \frac{1}{2\alpha\sqrt{M}}\right)\ \mathrm{erf}(\sqrt{Mt})$$

$$+\frac{1}{\alpha}\sqrt{\frac{t}{\pi}}\ e^{-Mt} + \frac{\sqrt{M-\alpha}}{\alpha^2}\ e^{-\alpha t}\ \mathrm{erf}(\sqrt{(M-\alpha)t}), \tag{40c}$$

$$F_4(t) = -\frac{\sqrt{-b\xi}}{\xi^2}\ e^{-\xi t}\ \mathrm{erf}(\sqrt{-\xi t}) - \frac{2}{\xi}\sqrt{\frac{bt}{\pi}}$$

$$+\left(\frac{\xi t-1}{\xi^2}\sqrt{M} + \frac{1}{2\xi\sqrt{M}}\right)\ \mathrm{erf}(\sqrt{Mt})$$

$$+\frac{1}{\xi}\sqrt{\frac{t}{\pi}}\ e^{-Mt} + \frac{\sqrt{M-\xi}}{\xi^2}\ e^{-\xi t}\ \mathrm{erf}(\sqrt{(M-\xi)t}), \tag{40d}$$

$$F_5(t) = \frac{\sqrt{-a\alpha}}{\alpha}\ e^{-\alpha t}\ \mathrm{erf}(\sqrt{-\alpha t}) + \frac{\sqrt{M}}{\alpha}\ \mathrm{erf}(\sqrt{Mt})$$

$$-\frac{\sqrt{M-\alpha}}{\alpha}\ e^{-\alpha t}\ \mathrm{erf}(\sqrt{(M-\alpha)t}), \tag{40e}$$

and
$$F_6(t) = \frac{\sqrt{-b\xi}}{\xi}\ e^{-\xi t}\ \mathrm{erf}(\sqrt{-\xi t}) + \frac{\sqrt{M}}{\xi}\ \mathrm{erf}(\sqrt{Mt})$$

$$-\frac{\sqrt{M-\xi}}{\xi}\ e^{-\xi t}\ \mathrm{erf}(\sqrt{(M-\xi)t}). \tag{40f}$$

6.3.2. *Movement of the plate with single acceleration.* When the plate moves with single acceleration, the skin friction for ramped temperature plate in the presence of ramped species concentration is expressed as

$$\tau_2 = -Q_2(t) - \alpha_1\{F_3(t) - H(t-1)F_3(t-1)\} - \alpha_2\{F_4(t) - H(t-1)F_4(t-1)\}, \tag{41}$$

and the skin friction for isothermal plate in the presence of uniform species concentration is obtained as

$$\tau_{2i} = -Q_2(t) - \alpha_1 F_5(t) - \alpha_2 F_6(t), \tag{42}$$

where,

$$Q_2(t) = -\left(\frac{1}{2\sqrt{M}} + \sqrt{M}\ t\right)\ \mathrm{erf}(\sqrt{Mt}) - \sqrt{\frac{t}{\pi}}\ e^{-Mt}. \tag{43}$$

6.3.3. *Oscillatory movement of the plate.* When the plate moves with periodic acceleration, the skin friction for ramped temperature plate in the presence of ramped species concentration is expressed as

$$\tau_3 = -Q_3(t) - \alpha_1\{F_3(t) - H(t-1)F_3(t-1)\} - \alpha_2\{F_4(t) - H(t-1)F_4(t-1)\}, \tag{44}$$

and the skin friction for isothermal plate in the presence of uniform species concentration is obtained as

$$\tau_{3i} = -Q_3(t) - \alpha_1 F_5(t) - \alpha_2 F_6(t), \tag{45}$$

where,

$$\begin{aligned} Q_3(t) = &-\frac{1}{2}\sqrt{M - i\omega}\ e^{-i\omega t}\ \mathrm{erf}\left(\sqrt{(M - i\omega)t}\right) \\ &-\frac{1}{2}\sqrt{M + i\omega}\ e^{i\omega t}\ \mathrm{erf}\left(\sqrt{(M + i\omega)t}\right) - \frac{e^{-Mt}}{\sqrt{\pi t}}. \end{aligned} \tag{46}$$

7. Numerical results and discussion

To study the effects of various flow parameters on the unsteady MHD free convective heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid past a flat plate with ramped wall temperature in the presence of ramped species concentration, under the influence of a transverse magnetic field. The profiles of species concentration $C(\eta, t)$, for different values of t and Sc, the profiles of fluid temperature $T(\eta, t)$, for various values of t, Pr and N, and the profiles of fluid velocity $u(\eta, t)$, for different values of t, Sc, Pr, N, M, Gr, and Gm, are presented in figs. 1-9 taking $\omega = \frac{\pi}{2}$. The velocity profiles are presented for various time dependent movement of the plate i.e. (a) uniform movement, (b) Impulsive movement and (c) oscillatory movement in the figs.3-9 with the fixed values of $t = 0.7$,

$Sc = 0.6$, $Pr = 0.71$, $N = 1$, $M = 6$, $Gr = 2$ and $Gm = 2$. In figs. 1-9, the solid lines denote the profiles of flow past a ramped temperature plate in the presence of ramped species concentration while the dotted lines represent the profiles in case of a flow past an isothermal plate in the presence of uniform species concentration. The numerical values of skin friction, Nusselt number and Sherwood number are presented in tables 1-4 for various values of pertinent flow parameters taking $\omega = \frac{\pi}{2}$ for both the cases.

It is noticed from figs. 1 (a) & (b) that, the species concentration $C(\eta, t)$ increases with an increase in t and decreases with an increase in Sc. Since, the Schmidt number Sc is the ratio of viscosity to mass diffusivity, an increase in Sc implies a decrease in mass diffusion rate. Thus it follows that the species concentration increases with an increase in time or mass diffusion rate. It is observed from figs.2 (a), (b) & (c) that, for both ramped temperature and isothermal plates, the fluid temperature $T(\eta, t)$ increases with an increase in t or N and decreases with an increase in Pr. Since the Prandtl number Pr is the ratio of viscosity to the thermal diffusivity, an increase in Pr implies a decrease in thermal diffusivity. This implies that the time, thermal diffusion and radiation tend to increase the fluid temperature.

It is revealed from figs. 3 (a), (b) & (c) that, for both the cases, fluid velocity $u(\eta, t)$ increases with an increase in t in all the particular cases except in the case of periodic acceleration of the plate. In this case the fluid velocity, near the plate, decreases with an increase in t and afterward it attains its usual nature which is to increase with an increase in t. Thus, in general, it may be concluded that the fluid velocity in all the three particular cases of interest, increases with increase in time. It is noticed from figs. 4-9 that, the fluid velocity decreases with an increase in Sc, Pr, or M whereas it increases with an increase in N, Gr or Gm. Thus it follows that the magnetic field tends to retard the fluid flow whereas mass diffusion, thermal diffusion, radiation, thermal buoyancy force and mass buoyancy force have reverse effect on it. It is also noticed from figs. 3-9, that the fluid velocity, in case of a flow past an isothermal plate in the presence of uniform species concentration, is higher than that of a flow past a ramped temperature plate in the presence of ramped species concentration.

It is observed from tables 1-3 that, the skin friction, for both the cases, in all the particular cases, decreases with an increase in time t except for the case where the plate moves with single acceleration. In this case the skin friction, for both the cases, increases with an increase in time. It is also evident from these three tables that the skin friction for the case of flow past a ramped temperature plate in the presence of ramped species concentration, in all the three particular cases, increases with an increase in Sc whereas it decreases with an increase in N. Which implies that, in case of a flow past a ramped temperature plate and ramped species concentration, mass diffusion and radiation tend to decrease the skin friction. It is interesting to note that the mass diffusion and radiation have no effect on the skin friction in case of a flow past an isothermal plate in the presence of uniform species concentration. Further, it is noticed that, for both cases, the skin friction, in all the three particular cases, increases with an increase in Pr or M whereas it decreases with an increase in Gr or Gm. Thus, it may be concluded that, the magnetic field tends to increase the skin friction whereas thermal diffusion, thermal buoyancy force and mass buoyancy force have reverse effect on it.

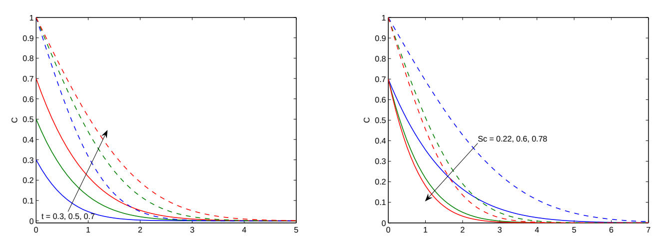
It is noticed from table 4 that, the Nusselt number, for the ramped temperature plate, increases with an increase in time t but for isothermal plate, it decreases with an increase in t. It is noticed from the table that the Nusselt number for both ramped temperature plate and isothermal plate, increases with an increase in Pr, whereas it decreases with an increase in N which implies that the thermal diffusion and radiation tend to decrease skin friction. It is also observed from table 4 that the Sherwood number for ramped species concentration increases with an increase in t whereas, for uniform species concentration, decreases with an increase in t. It is noticed from table 4 that, for both ramped and uniform species concentration, the Sherwood number increases with an increase in Sc. Thus it follows that, the mass diffusivity tends to reduce the rate of mass transfer at the plate whereas the rate of mass transfer, for ramped species concentration increases, while for uniform species concentration, it decreases with the passage of time.

8. Conclusions

An investigation of unsteady MHD free convective heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat radiating fluid past an infinite flat plate with ramped wall temperature in the presence of ramped species concentration, under the influence of a transverse magnetic field is carried out. The flow is induced due to a time dependent movement of the flat plate. Three cases of particular interest were discussed viz. (1) movement of the plate with uniform velocity, (2) movement of the plate with single acceleration and (3) oscillatory movement of the plate.
The important findings of the present work are summarized below.

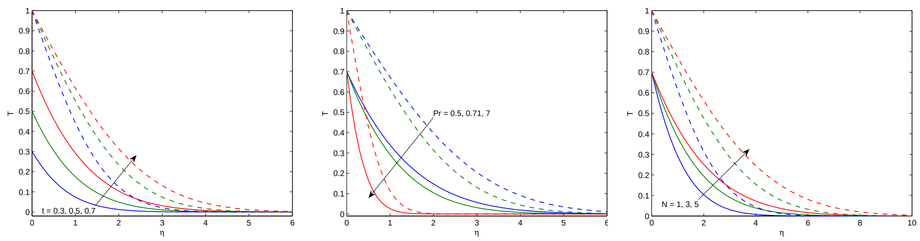
- Mass diffusion rate and time tend to increase the species concentration.
- Time, thermal diffusion and radiation tend to increase the fluid temperature.

- Magnetic field tends to retard the fluid flow whereas mass diffusion, time, thermal diffusion, radiation, thermal buoyancy force and mass buoyancy force have reverse effect on it.
- Magnetic field tends to increase the skin friction whereas thermal diffusion, thermal buoyancy force and mass buoyancy force have reverse effect on it.
- Thermal diffusion and radiation tend to decrease skin friction.
- Mass diffusivity tends to reduce the rate of mass transfer at the plate whereas the rate of mass transfer, for ramped species concentration increases, while for uniform species concentration it decreases with the passage of time.



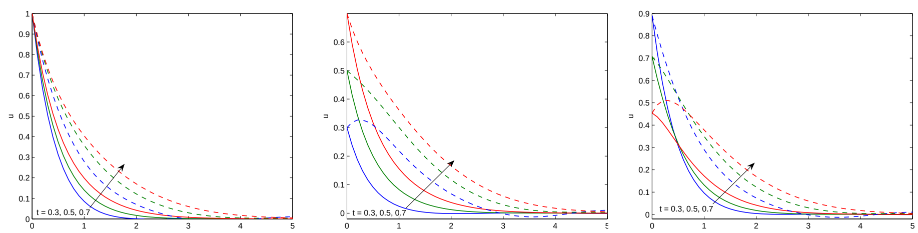
(a) (b)

FIGURE 1. Species concentration profiles of the fluid when (a) $Sc = 0.6$, (b) $t = 0.7$.



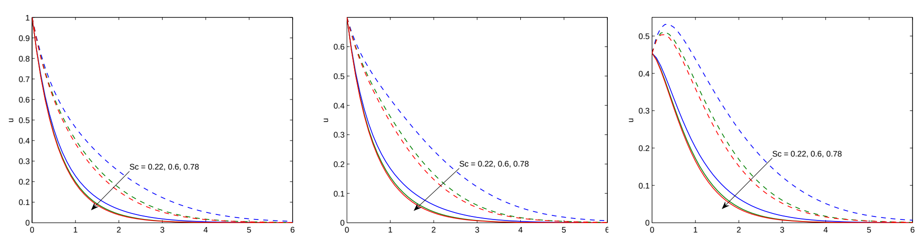
(a) (b) (c)

Figure 2. Variation of time, Prandtl number & thermal radiation on temperature profile.



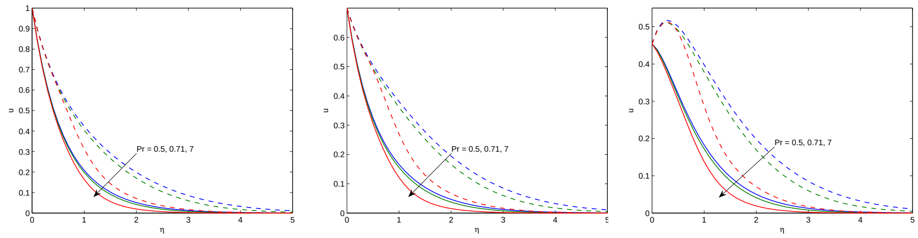
(a) (b) (c)

Figure 3. Variation of time on velocity profile of the fluid.



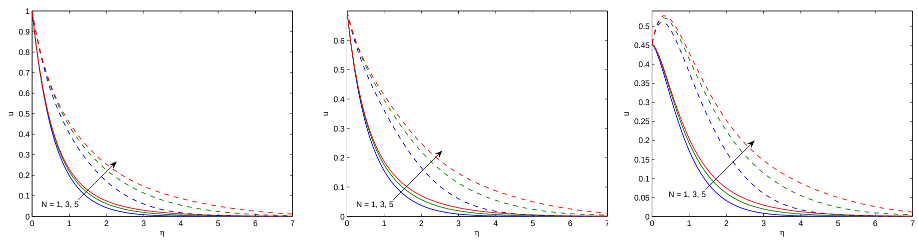
(a) (b) (c)

Figure 4. Variation of Sc on velocity profile of the fluid.



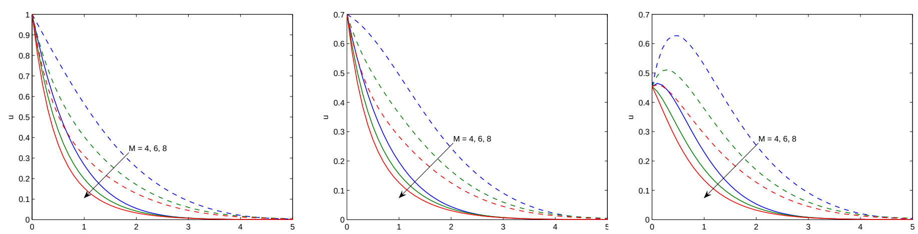
(a) (b) (c)

FIGURE 5. Variation of Pr on velocity profile of the fluid.



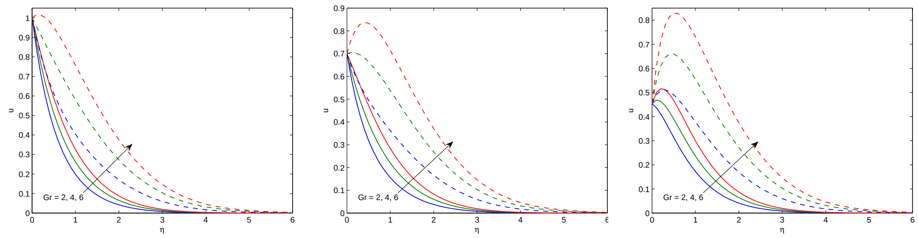
(a) (b) (c)

FIGURE 6. Variation of N on velocity profile of the fluid.



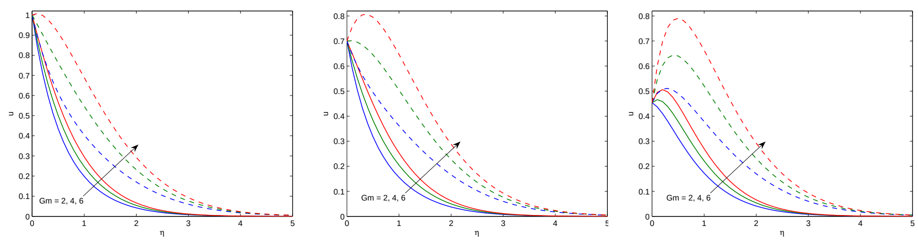
(a) (b) (c)

FIGURE 7. Variation of M on velocity profile of the fluid.



(a) (b) (c)

FIGURE 8. Variation of Gr on velocity profile of the fluid.



(a) (b) (c)

FIGURE 9. Variation of Gm on velocity profile of the fluid.

TABLE 1. Skin friction for different values of the governing parameters when $f(t) = H(t)$.

t	Sc	Pr	N	M	Gr	Gm	τ_1	τ_{1i}
0.3	0.6	0.71	1	6	2	2	2.27067541	0.93958828
0.5	-	-	-	-	-	-	2.00662854	0.84454021
0.7	-	-	-	-	-	-	1.74424242	0.82354496
-	**0.22**	-	-	-	-	-	1.69960473	0.82354496
-	**0.6**	-	-	-	-	-	1.74424242	0.82354496
-	**0.78**	-	-	-	-	-	1.75400828	0.82354496
-	-	**0.5**	-	-	-	-	1.72832430	0.82354496
-	-	**0.71**	-	-	-	-	1.74424242	0.82354496
-	-	**7**	-	-	-	-	1.86206693	1.16666375
-	-	-	**1**	-	-	-	1.74424242	0.82354496
-	-	-	**3**	-	-	-	1.71338954	0.82354496
-	-	-	**5**	-	-	-	1.69708178	0.82354496
-	-	-	-	**4**	-	-	1.25945745	0.04100639
-	-	-	-	**6**	-	-	1.74424242	0.82354496
-	-	-	-	**8**	-	-	2.16601081	1.41555037
-	-	-	-	-	**2**	-	1.74424242	0.82354496
-	-	-	-	-	**4**	-	1.37963602	0.01011205
-	-	-	-	-	**6**	-	1.01502962	-0.80332086
-	-	-	-	-	-	**2**	1.74424242	0.82354496
-	-	-	-	-	-	**4**	1.40268045	0.01011205
-	-	-	-	-	-	**6**	1.06111847	-0.80332086

TABLE 2. Skin friction for different values of the governing parameters when $f(t) = tH(t)$.

t	Sc	Pr	N	M	Gr	Gm	τ_2	τ_{2i}
0.3	0.6	0.71	1	6	2	2	0.72824589	-0.60284124
0.5	-	-	-	-	-	-	0.98074652	-0.18134180
0.7	-	-	-	-	-	-	1.21247741	0.29177996
-	**0.22**	-	-	-	-	-	1.16783972	0.29177996
-	**0.6**	-	-	-	-	-	1.21247741	0.29177996
-	**0.78**	-	-	-	-	-	1.22224327	0.29177996
-	-	**0.5**	-	-	-	-	1.19655929	0.29177996
-	-	**0.71**	-	-	-	-	1.21247741	0.29177996
-	-	**7**	-	-	-	-	1.33030192	0.63489875
-	-	-	**1**	-	-	-	1.21247741	0.29177996
-	-	-	**3**	-	-	-	1.18162453	0.29177996
-	-	-	**5**	-	-	-	1.16531677	0.29177996
-	-	-	-	**4**	-	-	0.90344170	-0.31500936
-	-	-	-	**6**	-	-	1.21247741	0.29177996
-	-	-	-	**8**	-	-	1.49406076	0.74360033
-	-	-	-	-	**2**	-	1.21247741	0.29177996
-	-	-	-	-	**4**	-	0.84787101	-0.52165296
-	-	-	-	-	**6**	-	0.48326461	-1.33508587
-	-	-	-	-	-	**2**	1.21247741	0.29177996
-	-	-	-	-	-	**4**	0.87091544	-0.52165296
-	-	-	-	-	-	**6**	0.52935346	-1.33508587

TABLE 3. Skin friction for different values of the governing parameters when $f(t) = \cos\omega t H(t)$.

t	Sc	Pr	N	M	Gr	Gm	τ_3	τ_{3i}
0.3	0.6	0.71	1	6	2	2	1.88929567	0.55820855
0.5	-	-	-	-	-	-	1.08600528	-0.07608305
0.7	-	-	-	-	-	-	0.13703796	-0.78365950
-	**0.22**	-	-	-	-	-	0.09240027	-0.78365950
-	**0.6**	-	-	-	-	-	0.13703796	-0.78365950
-	**0.78**	-	-	-	-	-	0.14680382	-0.78365950
-	-	**0.5**	-	-	-	-	0.12111984	-0.78365950
-	-	**0.71**	-	-	-	-	0.13703796	-0.78365950
-	-	**7**	-	-	-	-	0.25486247	-0.44054071
-	-	-	**1**	-	-	-	0.13703796	-0.78365950
-	-	-	**3**	-	-	-	0.10618508	-0.78365950
-	-	-	**5**	-	-	-	0.08987732	-0.78365950
-	-	-	-	**4**	-	-	-0.13943626	-1.35788732
-	-	-	-	**6**	-	-	0.13703796	-0.78365950
-	-	-	-	**8**	-	-	0.38247881	-0.36798162
-	-	-	-	-	**2**	-	0.13703796	-0.78365950
-	-	-	-	-	**4**	-	-0.22756844	-1.59709241
-	-	-	-	-	**6**	-	-0.59217484	-2.41052532
-	-	-	-	-	-	**2**	0.13703796	-0.78365950
-	-	-	-	-	-	**4**	-0.20452402	-1.59709241
-	-	-	-	-	-	**6**	-0.54608599	-2.41052532

TABLE 4. Nusselt number and Sherwood number for different parameters governing the model.

t	Sc	Pr	N	Nu	Nu_i	Sh	Sh_i
0.3	0.6	0.71	1	0.36823907	0.61373178	0.47873074	0.79788456
0.5	0.6	0.71	1	0.47539459	0.47539459	0.61803872	0.61803872
0.7	0.6	0.71	1	0.56249447	0.40178176	0.73127328	0.52233806
0.7	**0.22**	0.71	1	-	-	0.44280796	0.31629140
0.7	**0.6**	0.71	1	-	-	0.73127328	0.52233806
0.7	**0.78**	0.71	1	-	-	0.83377982	0.59555702
0.7	0.6	**0.5**	1	0.47203487	0.33716777	-	-
0.7	0.6	**0.71**	1	0.56249447	0.40178176	-	-
0.7	0.6	**7**	1	1.76619277	1.26156626	-	-
0.7	0.6	0.71	**1**	0.56249447	0.40178176	-	-
0.7	0.6	0.71	**3**	0.39774365	0.28410261	-	-
0.7	0.6	0.71	**5**	0.32475633	0.23196881	-	-

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