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On Sequence Spaces and Some Matrix Transformations

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Abstract

The goal of this paper is to characterize the matrix transformations and is related to the concept of invariant mean and the lacunary sequence.

Key words: lacunary sequence, sequence space, invariant means, matrix transformations.
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1. Introduction and Preliminaries

We shall write w for the set of all complex sequences $x = (x_k)_{k=0}^{\infty}$. Let ϕ, l_∞, c and c_0 denote the sets of all finite, bounded, convergent and null sequences respectively. We write $l_p = \{x \in w : \sum_0^\infty |x_p| < \infty\}$ for $1 \leq p < \infty$. We denote the sequences $e = (1, 1, 1, ...)$ and $e^n = (0, 0, 0, ..., 1(nthplace), 0, ...)$. For any sequence $x = (x_k)_{k=0}^{\infty}$, we denote the n-section by $x^{[n]} = \sum_{k=0}^{n} x_k e^{(k)}$. Note that l_∞, c and c_0 Banach spaces with the sup-norm $||x||_\infty = \sup_k |x_k|$, and $l^p (1 \leq p < \infty)$ are Banach spaces with the norm $||x||_p = (\sum |x_k|^p)^{1/p}$; while ϕ is not a Banach space with respect to any norm.

Schaefer [25] has defined the concepts of σ-conservative, σ-regular and σ-coercive matrices and characterized matrix classes (c, V_σ), $(c, V_\sigma)_{reg}$ and (l_∞, V_σ), where V_σ denote the set of all bounded sequences all of whose invariant means (or σ-means) are equal. Recently, in [9] and [10], Mursaleen characterized some matrix classes by using de la Valée-poussin and invariant mean. Matrix transformations between sequence spaces have been discussed by Savaş and Mursaleen [23], Başarir and Savaş [2], Nanda [12], Nanda and Bilgin [13], Vatan [5], Vatan and Simşek [6], Savaş ([16], [17], [18], [19], [20],[21]) and many others.

Let σ be a mapping of the set of positive integers into itself. A continuous linear functional ϕ on l_∞, the space of real bounded sequences, is said to be an invariant mean or a σ-mean if and only if (1)$\phi(x) \geq 0$ when the sequence $x = (x_n)$ has $x_n \geq 0$ for all n, (2) $\phi(e)=1$, where $e=(1,1,...)$ and (3) $\phi(x(\sigma(n))) = \phi(x)$ for all $x \in l_\infty$. Throughout the paper, for typographical convenience we shall use the notation $x(\sigma(n)))$ to denote $x_\sigma(n)$.
The mappings σ are one-to-one and such that $\sigma^m(n) \neq n$, for all positive integers n and m, where $\sigma^m(n)$ denotes the mth iterate of the mappings σ at n. Thus σ-extends the limit functional on c, the space of convergent sequences, in the sense that $\phi(x) = limx$ for all $x \in c$. Consequently, $c \subset V_\sigma$ where V_σ is the set of bounded sequence all of whose σ-means are equal.
In case σ is the translation mapping $n \to n{+}1$, a σ-mean is often called a Banach limit (see, [1]) and V_σ is the set of almost convergent sequences.
If $x = (x_n)$, set $Tx = (Tx_n) = (x(\sigma(n)))$. It can be shown(see, [25])

$$V_\sigma = \{x \in l_\infty : \lim_m t_{mn}(x) = L, \quad uniformly \quad in \quad n, \quad L = \sigma - limx\}... \tag{1.1}$$

where

$$t_{mn}(x) = \frac{1}{m+1}\sum_{k=0}^{m} x(\sigma^k(n)).$$

and $t_{-1}, n(x) = 0$.
The special case of (1.1) in which $\sigma(n) = n + 1$ was given by Lorentz [7].

By a lacunary $\theta = (k_r)$; $r = 0, 1, 2, ...$ where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and $h_r = k_r - k_{r-1}$. The ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r. Freedman at al [4] defined the space of lacunary strongly convergent sequences N_θ as follows:

$$N_\theta = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} |x_k - le|) = 0, \text{ for some } l \right\}.$$

There is a strong connection between N_θ and the space w of strongly Cesàro summable sequences which is defined by Maddox [8] as follows;

$$w = \left\{ x = (x_k) : \lim_n \frac{1}{n} \sum_{k=0}^{n} |x_k - le|) = 0, \text{ for some } l \right\}.$$

In the special case where $\theta = (2^r)$, we have $N_\theta = \sigma$.

Quite recently, concept of lacunary σ-convergent was introduced and studied by Savas [22] which is a generalization of the idea of lacunary almost convergence due to Das and Mishra [3]. If $x \in V_\sigma^\theta$ denotes the set of all lacunary σ-convergent sequences, then Savas [22] defined

$$V_\sigma^\theta = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} (x(\sigma^k(n))) - L) = 0, \text{for some} L, \text{ uniformly in } n \right\}.$$

Note that for $\sigma(n) = n + 1$, the space V_σ^θ is the same as AC_θ. We write $V_\sigma^\theta = V_{\sigma_0}^\theta$ whenever $L = 0$.Then,

$$V_\sigma^\infty(\theta) := \{x \in l_\infty : \sup_{r,n} |t_{rn}(x) \leq \infty\},$$

where

$$t_{nr}(x) = \frac{1}{h_r} \sum_{k \in I_r} x(\sigma^k(n)).$$

If $\theta = 2^r$ and $\sigma(n) = n + 1$, then $V_\sigma^\infty(\theta)$ is reduced to the set f_∞ defined by Nanda [11].

Just as boundedness is related to convergence, it is quite natural to expect that the sequence space $V_\sigma^\infty(\theta)$ is related to $\sigma-$ convergence. But we observe that this concept coincide with l_∞. To prove this let $x \in V_\sigma^\infty(\theta)$. Then there is a constant $M > 0$ such that

$$\frac{1}{h_1} |x(\sigma^1(n)| \leq \sup_{r,n} \frac{1}{h_r} \sum_{k \in I_r} |x(\sigma^k(n))| \leq M$$

for all n and so $x \in l_\infty$. Conversely, let $x \in l_\infty$. Then there is a constant $M > 0$ such that $|x_j| \leq M$ for all j and so

$$\frac{1}{h_r} \sum_{k \in I_r} |x(\sigma^k(n))| \leq M \frac{1}{h_r} \sum_{k \in I_r} 1 \leq M$$

for all r and n and so $x \in V_\sigma^\infty(\theta)$. Therefore $V_\sigma^\infty(\theta) = l_\infty$.

The space $V_\sigma(\theta)$ is BK spaces with the norm $||x|| = \sup_{r,n} |t_{rn}(x)|$. In this paper we characterize matrix classes by using lacunary sequence space such as $(l_p, V_\sigma^\infty(\theta))$ and $(l_p, V_\sigma(\theta))$.

2. Main Results

Let X and Y be two sequence spaces, $B = (b_{nk})_{n;k=1}^\infty$ be an infinite matrix of real or complex numbers and $B_n = (b_{nk})_{k=1}^\infty$ be the sequence in the n-th row of B. We write $Bx = B_n(x)$, where $B_n(x) = \sum_k b_{nk} x_k$ provided that the series on the right converges for each n. If $x = (x_k) \in X$, implies that $Bx \in Y$, then we say that A defines a matrix transformation from$X$ into Y and by (X, Y) we denote the class of such matrices, that is, $B \in (X, Y)$ if and

only if $B_n \in X^\beta$ for all n and $Bx \in Y$ for all $x \in X$.
Let Bx be defined. Then, for all r, n, we write

$$t_{nr}(Bx) = \sum_{k=1}^{\infty} t(n,k,r)x_k \quad ,$$

where

$$t(n,k,r) = \frac{1}{h_r}\sum_{i\in I_r} b(\sigma^i(n),k),$$

and $b(n,k)$ denotes the element b_{nk} of the matrix B.

Let $X(p)$ denote the set of all sequences $x = (x_k)$ such that the following norms are finite:

$$\|x\|_{X(p)} = \left\{\sum_{s=0}^{\infty}\left|x_s^{'}\right|^p\right\}^{1/p}, \text{ for } 1 \leq p < \infty$$

and

$$\|x\|_{X(\infty)} = \sup\left\{\left|x_s^{'}\right|; s \geq 0\right\},$$

where

$$x_s^{'} = 2^{-s}\sup\left\{\left|\sum_{i=2^s}^{k} x_i\right|; 2^s \leq k < 2^{s+1}\right\}.$$

To simplify our presentation we shall confine ourselves to $1 < p < \infty$.
Next let $Y(q)$ denote the set of all sequences $y = (y_k)$ such that the following norms are finite;

$$\|x\|_{Y(q)} = \{\sum_{s=0}^{\infty}|y_s^{'}|^q\}^{1/q}, \text{ for } 1 < q < \infty$$

where

$$y_s^{'} = 2^s\{\sum_{2^s=k<2^{s+1}-1}|y_k - y_{k+1}| + |y_{2^{s+1}-1}|\}$$

The cases where $q = 1$ and $q = \infty$ are similar. In what follows we shall always assume $\frac{1}{p} + \frac{1}{q} = 1$

We now obtain the following theorem

Theorem 2.1. *$A \in (X(p), V_\sigma(\theta))$ if and only if*

(i) $M = sup\{\|b(n,k,r)_{k\geq 1}\|_{Y(q)}; m \geq 1\} < \infty,$ *and*

(ii) $lim_r b(n,k,r) = \alpha_k$ *uniformly in* n *, (k fixed)*

Proof. The necessity is open. To prove the sufficiency given $x \in X(p)$ we want to show that Ax belongs to $V_\sigma(\theta)$. First we observe that $\alpha = (\alpha_k) \in Y(q)$ and $\|\alpha\|_{Y(q)} \leq M$ where M is the constant. Since $x \in X(p)$ for any given $\varepsilon > 0$, we can choose r_0 such that

$$\{\sum_{s=s_0+1}^{\infty} |x_s^{'}|^p\}^{1/p} < \frac{\varepsilon}{4M}.$$

Then we can find that for sufficiently large n

$$\begin{aligned}
\Big|\sum_{k=1}^{\infty}(b(n,k,r) - \alpha_k)x_k\Big| & \\
&\leq \Big|\sum_{s=0}^{\infty}\Big|\sum_{s}(b(n,k,r) - \alpha_k)x_k\Big| \\
&\leq \sum_{s=0}^{s_0}\Big|\sum_{s}(b(n,k,r) - \alpha_k)x_k\Big| + \sum_{s=s_0+1}^{\infty}\Big|\sum_{s}(b(n,k,r) - \alpha_k)x_k\Big| \\
&\leq \frac{\varepsilon}{2} + 2M.\frac{\varepsilon}{4M} \\
&= \varepsilon
\end{aligned}$$

□

Hence the proof is completed.

Let us denote v the space of sequences of bounded variation, that is

$$v = \{x : \sum_k \mid x_k - x_{k-1} \mid < \infty, x_0 = 0\}$$

v is a Banach space normed by $\|x\| = \sum_k \mid x_k - x_{k-1} \mid$.

We have

Theorem 2.2. *$A \in (v, V_\sigma(\theta)$ if and only if*

(i)

$$M = sup_r |\sum_{k=t}^{\infty} b(n,k,r)| < \infty, t, n = 1, 2, ...$$

(ii) there exists an $\alpha \in C$ such that

$$lim_n \sum_k b(n,k,r) = \alpha,$$

uniformly in n ,

and

(iii) there exists an $\alpha_k \in \boldsymbol{C}(k = 0, 1, 2, ...)$ such that

$$lim_r b(n,k,r) = \alpha_k,$$

uniformly in n.

Proof. Suppose that $B \in (v, V_\sigma(\theta))$. This implies that $Bx \in V_\sigma(\theta)$ for $x \in v$. Since $V_\sigma(\theta) \subset \ell_\infty$,

$Bx \in \ell_\infty$ and hence (i) holds. Define $e_k = (0, 0, ..., 0, 1(\text{kth place}), 0, ...)$and $e = (1, 1, ...)$. Since e_k and e are in v , (ii) and (iii) must hold.

Conversely, suppose that the conditions (i) - (iii) hold and $x \in v$. Since $v \subset$ c, therefore $x_k \quad \to \ell$. Now

$$\sum_k \Big|b(n,k,r)x_k\Big| \leq \quad \sum_k |x_k - x_{k-1}|\Big|\sum_{k=1}^{t} b(n,k,r)\Big| + \ell\Big|\sum_k b(n,k,r)\Big|.$$

By (i) and (iii) we get for each r,

$$sup_t\Big|\sum_{k=1}^{t} b(n,k,r)\Big| < \infty.$$

Therefore $t_{nr}(Bx)$ exists for each n and $x \in v$. Also $\sum \alpha_k x_k$ exists for each $x \in v$. For given $\varepsilon > 0$, choose and fix $k_0 \in Z^+$ such that

$$\sum_{k=k_0+1} |x_k - x_{k-1}| < \varepsilon/4M.$$

We have

$$|t_{nr}(Bx) - \sum_k \alpha_k x_k - \ell \sum_k (b(n,k,r) - \alpha_k)| \leq I_1 + I_2$$

where

$$I_1 = \sum_{k=1}^{k_0}\Big|\sum_{k=1}^{t}(b(n,k,r) - \alpha_k)\Big|\Big|x_k - x_{k-1}\Big|,$$

and

$$I_2 \leq sup_t\Big|\sum_{k=1}^{t}(b(n,k,r) - \alpha_k)\Big|\sum_{k=k_0+1}|x_k - x_{k-1}\Big|.$$

By virtue of condition (iii) there exists an integer $n_o > 0$ such that $I_1 \quad \leq \varepsilon/2$ for $n \geq n_o$. Clearly $I_2 \quad \leq \quad \varepsilon/2$. Further by virtue of condition (ii) we have for $n \geq n_o$,

$$\Big|t_{nr}(Bx) - \sum_k \alpha_k x_k - \ell(\alpha - \sum_k \alpha_k)\Big| \leq \quad \varepsilon,$$

therefore we have uniformly in n,

$$lim_r t_{nr}(Bx) = \ell\alpha + \sum_k \alpha_k(x_k - \ell),$$

so that $Bx \in V_\sigma(\theta)$ and this completes proof. □

We write $(v, V_\sigma(\theta), P)$ to denote the subset of $(v, V_\sigma(\theta))$ such that A is almost lacunary convergent to the limit of x in v.

We know consider the class $(v, V_\sigma(\theta), P)$.

Theorem 2.3. *$A \in (v, V_\sigma(\theta), P)$ if and only if*

(iv) the condition (i) of Theorem 2.2 holds

(v) $lim_r \sum_k b(n,k,r) = 1$, uniformly in n ,

and

(vi) $lim_r b(n,k,r) = 0$, for each k uniformly in n.

Proof. Let $B \in (v, V_\sigma(\theta), P)$. Then conditions hold by theorem 2.2. Let the conditions (i)-(iii) hold. Then by Theorem 2.2., $B \in (v, V_\sigma(\theta))$ and

$$lim_r \sum_k b(n,k,r)x_k = \ell,$$

uniformly in n.

This completes the proof. □

The following sequence space has been defined in [15].

Let $X_p (1 \leq p < \infty)$ be the space of all $x \in X$ with

$$\|x\|_p = \Big(\sum_{n=1}^{\infty} \Big|\frac{1}{n}\sum_{k=1}^{n} x_k\Big|^p\Big)^{1/p}$$

for $1 \leq p < \infty$.

It is easy to see that $X_p (1 \leq p < \infty)$ is a Banach space of nonabsolute type and the above norm is saturated except for $p = 1$,(see, [15]).
Let Y_q be the space of all $y \in$Y such that

(a) $|ky_k| \leq M$ for all $k = 1, 2, \ldots$

(b) $\alpha_q(y) = \left(\sum_{k=1}^{\infty} |k(y_k - y_{k+1})|^q \right)^{1/q} < \infty$ for $1 \leq q < \infty$

and $\quad \alpha_\infty(y) = sup \{|k(y_k - y_{k+1})|\, ; k = 1, 2, \ldots\} < \infty$.

The following theorem is due to Ng [14].

Theorem 2.4. *The associate space $X_{p'}$ of X_p is the space Y_q with the norm α_q, where $\frac{1}{p} + \frac{1}{q} = 1$.*

We need the following lemma is due to [15] for the proof of the next theorem.

Lemma 2.1. *A matrix A transforms a BK- space E into a BK- space F then the transformation is linear and continuous.*

We now have

Theorem 2.5. *An infinite matrix $B \in (X_p, V_\sigma(\theta))$ if and only if B satisfies the following conditions :*

(i) $\quad sup_{nr} \left\| \{k(b(n,k,r) - b(n,k+1,r))\}_{k \geq 1} \right\|_q < \infty$,

(ii) $\quad sup_k |kb(n,k,r)| < \infty$ *for every fixed* n, r,

(iii) $\quad lim_r k(b(n,k,r) - b(n,k+1,r)) = \delta_k$,*uniformly in* n, *for every fixed* k,

where $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. First we prove that the conditions are necessary . Suppose $B = (b_{nk})$ maps X_p into $V_\sigma(\theta)$, then the series

$$t_{nr}(Bx) = \sum_{k=1}^{\infty} b(n,k,r)x_k$$

is convergent for every n, r and for every $x \in X_p$. Then Theorem 2.4 the sequence $(b(n,k,r))_k$is an element in Y_q for every n, r. It follows that the condition (ii) holds and

$$\|\{k(b(n,k,r) - b(n,k+1,r))\}_{k\geq 1}\| < \infty.$$

Since X_p and $V_\sigma(\theta)$ are BK-spaces therefore by Lemma 2.1, we have

$$\|t_{nr}(Bx)\| \leq K \|x\|_p$$

for some real constant K, and all $x \in X_p$ or

$$sup_{nr} |t_{n,r}(Bx)| \leq K \|s\|$$

for all $x \in X_p$ with $s = (s_k)$ where

$$s_k = \frac{1}{k} \sum_{i=1}^{k} x_i.$$

It follows that

$$\sup_{nr} \left| \frac{\sum_{k=1}^{\infty} k(b(n,k,r) - b(n,k+1,r))s_k}{\|s\|} \right| \leq K.$$

Hence we have

$$sup_{n,r} \|\{k(b(n,k,r) - b(n,k+1,r))\}_{k\geq 1}\| \leq K.$$

Therefore the condition (i) holds. To prove the condition (iii) is necessary . We take for each fixed k, a sequence $x^{(k)}$ in X_p with $x_j^{(k)} = k$, if $j = k$, $-k$ if $j = k+1$, =0, if $j \neq k, k+1$. Then we see that

$$s_k = \frac{1}{k} \sum_{k=1}^{k} x_j^{(k)} = 1$$

and $s_j = 0$ if $j \neq k$. For this $x^{(k)}$ we have,

$$t_{rn}(Bx^{(k)}) = \sum_{j=1}^{\infty} b(n,j,r)x_j^{(k)} = \sum_{j=1}^{\infty} j(b(n,j,r) - b(n,j+1,r))s_j$$
$$= k(b(n,k,r) - (b(n,k+1,r) \to \delta_k$$

as $r \to \infty$, uniformly in r. This shows that condition (ii) is necessary.

Conversely, suppose the conditions (i), (ii) and (iii) hold. Then by conditions (i) and (ii) the series

$$t_{rn}(Bx) = \sum_{k=1}^{\infty} b(n,k,r)x_k$$

is convergent for every n, r and $x \in X_p$. By the condition (iii) we have

$$|k(b(n,k,r) - b(n,k+1,r))|^q \to \|\delta_k|^q$$

as $r \to \infty$ uniformly in n and since for every positive integer p

$$\left\{\sum_{k=1}^{p} |k(b(n,k,r) - b(n,k+1,r))|^q\right\}^{1/q} \le sup_{nr}\left\{\sum_{k=1}^{\infty} |k(b(n,k,r) - b(n,k+1,r))|^q\right\}^{1/q} = \beta$$

by letting $r \to \infty$ we get

$$\left\{\sum_{k=1}^{p} |\delta_k|^q\right\}^{1/q} \le sup_{r,n}\left\{\sum_{k=1}^{p} |k(b(n,k,r) - b(n,k+1,r))|^q\right\}^{1/q}.$$

Since this true for every positive integer p , it follows that

$$\left\{\sum_{k=1}^{p} |\delta_k|^q\right\}^{1/q} < \infty.$$

Now for every sequence $x \in X_p$, we have

$$s_n = \frac{1}{n}\sum_{k=1}^{n} x_k$$

as $n \to \infty$. Given any $\varepsilon > 0$, there exists $N > 0$ such that

$$\left\{\sum_{k=N}^{\infty}|s_k|^p\right\}^{1/p}<\frac{\varepsilon}{4\beta}.$$

And by condition (iii) there exists integer N_1 such that

$$\left|\sum_{k=1}^{N}\left\{k(b(n,k,r)-b(n,k+1,r)-\delta_k)\right\}s_k\right|<\frac{\varepsilon}{2},$$

for all $r>N_1$. Now for all $r>N_1$,

$$\begin{aligned}\left|\sum_{k=1}^{\infty}\left\{k(b(n,k,r)-b(n,k+1,r)-\delta_k)\right\}s_k\right| &\le \left|\sum_{k=1}^{N}\left\{k(b(n,k,r)-b(n,k+1,r)-\delta_k)\right\}s_k\right| \\ &\quad+\left|\sum_{k=N+1}^{\infty}\left\{k((b(n,k,r)-b(n,k+1,r)-\delta_k)\right\}s_k\right| \\ &< \frac{\varepsilon}{2}+\Big(\sum_{k=N+1}^{\infty}\left\{|k(b(n,k,r)-b(n,k+1,r)|+|\delta_k)|\right\}^q\Big)^{1/q}\times\Big(\sum_{k=N+1}^{\infty}|s_k|^p\Big)^{1/p} \\ &< \frac{\varepsilon}{2}+2\beta\frac{\varepsilon}{4\beta}=\varepsilon.\end{aligned}$$

So we have

$$lim_r\sum_{k=1}^{\infty}k(b(n,k,r)-b(n,k+1,r)-\delta_k)s_k=\sum_{k=1}^{\infty}\delta_k s_k$$

uniformly in n. It follows that

$$\begin{aligned}lim_r t_{nr}(Bx) &= lim_r\sum_{k=1}^{\infty}b(n,k,r)x_k \\ &= lim_r\sum_{k=1}^{\infty}k(b(n,k,r)-b(n,k+1,r))s_k \\ &= \sum_{k=1}^{\infty}\delta_k s_k\end{aligned}$$

uniformly in n. This show that $Bx\in V_\sigma(\theta)$ and $B=(b_{nk})$ maps $X_p,(1\le p<\infty)$ into $V_\sigma(\theta)$. This completes the proof. □

Corollary 2.1. *A matrix transformation $B = (b_{nk})$ maps the space X_p into the space $V_\sigma(\theta)_0$ if and only if*

(i) the conditions (i) and (ii) of Theorem 2.5 hold, b (ii) $lim_r k(b(n,k,r) - b(n,k+1,r)) = 0$, uniformly in n, for every fixed k, where $\frac{1}{p} + \frac{1}{q} = 1$.

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