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Modeling of Boundary Layer Two-Phase Flow and Heat Transfer over a Stretching Sheet due to Effects of Radiation, Heat Generation/Absorption and Electrification of Particles

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Abstract

In this paper, the effect of heat generation/absorption, radiation and electrification of particles on both phases of unsteady two phase flow over a stretching sheet has been investigated. The governing partial differential equations of the flow field are reduced into first order ordinary differential equations using similarity transformations and the solution is found by Runge-Kutta method with shooting technique. Comparison of the obtained results is made with existing literature and graphical study is performed to explain the inter relationship between parameters and velocity field, parameters and heat transfer characteristics. The rate of heat transfer at the surface and skin friction increases with increasing values of electrification parameter M. The temperature profile of both phases increase with the increase of radiation parameter Ra. Thus the radiation should be at minimum in order to facilitate the cooling process.

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Nomenclature:

x, y-Cartesian coordinates	P_r-Prandtl number
η -Similarity variable	F_r – Γroud number
u -Velocity component of fluid along x- axis	E_c -Eckert number
v -Velocity component of fluid along y-axis	oo -Density ratio
u -Velocity component of particle along x-axis	φ -Volume fraction
v -Velocity component of particle along y-axis	c -Stretching rate
$U_w(x)$ -Stretching sheet velocity	1 -Characteristic length
M– Electrification parameter	c– Specific heat of fluid
E -Electric field of force	c_s -Specific heat of particles
m -Mass of particle	k_s -Thermal conductivity of particle
e -Charge of particle	k -Thermal conductivity of fluid
T -Temperature of fluid phase.	A -Unsteady parameter
T -Temperature of particle phase.	a– Constant
T_w -Wall temperature	β – Γluid particle interaction parameter
T_∞-Temperature at large distance from the wall.	τ_T-Thermal relaxation time.
A^*andB^*- Non dimensional heat generation /absorption parameter for fluid phase	τ_p-Velocity relaxation time.
A_p^*andB_p^* -Non dimensional heat generation /absorption parameter for particle phase.	τ -Relaxation time of particle phase
6- Non-dimensional fluid phase temperature	μ -Dynamic viscosity of fluid
θ– Non-dimensional dust phase temperature	v -Kinematic viscosity of fluid
Ra -Radiation parameter	γ -Ratio of specific heat
q_r -Radiate heat flux for particle phase	ε– Diffusion parameter
q_r-Radiate heat flux for fluid phase	p -Density ofthe fluid phase
β^*-volumetric coefficient of thermal expansion	ρ_s– Material density
k^*-Mean absorption coefficient.	ρ -Density of the particle phase
σ^*– Stefan-Boltzman constant	

1 Introduction:

The flow over a stretching sheet has several industrial applications such as purification of crude oil, artificial fibers cooling industry of dying, cooling of nuclear reactor, aerospace component production, petroleum industry, glass blowing, cooling or drying of papers, drawing plastic films, extrusion of polymer melt-spinning process etc. The importance of such flow problems involving high temperature regime lies in the fact that the mechanical properties of final product are influenced by stretching rate and the rate of cooling. The rate of heat transfer over a surface has a pivotal role in the quality of final products. So the investigation of effects of radiation, heat generation/absorption and electrification ofparticles on the flow field and heat transfer analysis of unsteady two-phase flow over a stretching sheet is presented in this paper which will give a new dimension in the industrial sector for the progress ofthe society.

In 1961, Sakiadis [6] has first initiated the study of boundary layer flow over a stretched surface moving with a constant velocity. Then after, many researchers extended his study with the effect of heat transfer, of which, some of the important studies have cited bellow. Tsou.et.al [13] have studied the effects of heat transfer and experimentally confirmed the numerical results of Sakiadis. Chen [10] has investigated the mixed convection of a power law fluid past a stretching surface in presence ofthe thermal radiation and magnetic field. Crane [18] has obtained the exponential solutions for planar viscous flow of linear stretching sheet. The problem of two phase suspension flow is solved in the frame work of a two-way coupling model or a two-fluid approach. Grubka et.al[19] have interpreted the temperature field in the flow over a stretching surface when subject to uniform heat flux. Sharidan et.al. [37] have presented similarity solutions for unsteady boundary layer flow and heat transfer due to stretching sheet. Gireesha.et.al[9] have studied the effect of boundary layer flow and heat transfer of a dusty fluid over a vertical stretching surface. They have examined the heat transfer characteristics for two types of boundary conditions namely variable wall temperature and variable heat flux. Gireesh et.al [8] have also studied the mixed convective flow of a dusty fluid over a stretching sheet in presence ofthermal radiation and space dependent heat source/sink. Barik et.al [32] have studied the heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat sources. Mohammad et. a1.[25] have studied the heat transfer over an inclined stretching sheet in the presence of magnetic field. Sharma et.al [27] have investigated the momentum and heat transfer characteristics in MHD convective flow of dusty fluid over stretching sheet with heat source/sink.Soo[35] has studied the effect of electrification on the dynamics of a particulate system. Though many investigations have been made, but to the author's knowledge no study has been analyzed the effect of electrification of particles, particle-particle interactions, volume fraction and effect of diffusion parameter for fluid phase as well as particle phase. So our investigation will be a significant contribution to the literature which is not covered by the previous works. Here, it is mainly focused on the role ofthe inter particle electrostatic forces which have been given less attention by the previous investigators. At low temperature, electrification of solid particles occurs due to the impact of wall. Even a very small charge on the solid particles causes a pronounced effect on concentration distribution in the flow of a gas-solid system. Although electric charge on the solid particles can be excluded by definition in theoretical analysis or when dealt truly with a boundless system, electrification of the solid particles always occurs when it comes in contact and separation are made between the solid particles and a wall of different materials or similar materials but in different surface conditions. The electric charges on the solid particles cause deposition ofthe solid particles on a wall in a more significant manner than the gravity effect and are expected to affect the motion of a metalized propellant and its product of reaction through a rocket nozzle and the jet at the exit of the nozzle. The charged solid particles in thejet of a hot gas also affect radio communications. The above analysis has motivated to present this paper. Here the particles are allowed to diffuse through the carrier fluid. This can be done by applying the kinetic theory of gases as the motion of the particles across the streamline due to the concentration and pressure diffusion. Effects of Radiation, Heat Generation/Absorption, Electrification of Particles and other boundary layer characteristics on Unsteady Two-Phase Boundary

Schematic diagram of the flow

Layer Flow and Heat Transfer over a Stretching Sheet have been studied. The governing partial differential equations have reduced to a system of ordinary differential equations and solved with the help ofRunge-Kutta Method by using shooting techniques.

2 Modeling of the problem:

Here an unsteady two dimensional laminar boundary layer flow of viscous, incompressible dusty fluid over a vertical stretching sheet has been considered. The flow is caused by impermeable stretching sheet. The sheet is considered along the x-axis in which y-axis is taken as normal. Two equal and opposite forces are applied along the stretching sheet It is assumed that both the fluid and the dust particle clouds are stationary at the beginning as well as the dust particles are spherical in shape and uniform in size throughout the flow field.

The governing equations are;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial}{\partial t}\rho_p+\frac{\partial}{\partial x}(\rho_p u_p)+\frac{\partial}{\partial y}(\rho_p v_p)=0 \tag{2.2}$$

$$(1-\varphi)\rho[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(1-\varphi)\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\tau_p}\varphi\rho_s(u-u_p)+\varphi\rho_s(\frac{e}{m})E \tag{2.3}$$

$$\varphi\rho_s(\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y})=\frac{\partial}{\partial y}(\varphi\mu_s\frac{\partial u_p}{\partial y})+\frac{1}{\tau_p}\varphi\rho_s(u-u_p)+\varphi(\rho_s-\rho).g+\varphi\rho_s(\frac{e}{m})E \tag{2.4}$$

$$\varphi\rho_s(\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y})=\frac{\partial}{\partial y}(\varphi\mu_s\frac{\partial v_p}{\partial y})+\frac{1}{\tau_p}\varphi\rho_s(v-v_p) \tag{2.5}$$

With boundary conditions $u=U_w(x)=\frac{cx}{1-at}$, $v=0\ as\ y\to 0$

$\rho_p=\omega\rho, u=0$, $u_p=0$, $v_p\to v\ as\ y\to\infty$ Where ω is the density ratio in the main stream.

3 Flow Analysis:

Introducing the following non dimensional variables to solve equations (2.1) to (2.5)

$u=\frac{cx}{1-at}f'(\eta)$; $v=-\sqrt{\frac{cv}{1-at}}f(\eta)$;$\frac{\varphi\rho_s}{\rho}=\frac{\rho_p}{\rho}=\rho_r=H(\eta)$;$u_p=\frac{cx}{1-at}F(\eta)$;$v_P=\sqrt{\frac{cv}{1-at}}G(\eta)$; $\eta=\sqrt{\frac{c}{v(1-at)}}y$;$P_r=\frac{\mu c_p}{k}$; $\beta=\frac{1-at}{c\tau_p}$; $\varepsilon=\frac{v_s}{v}$; $\varphi=\frac{\rho_p}{\rho_s}$;A $=\frac{a}{c}$;$E_c=\frac{cv}{C_pT_0}$; $M=\frac{(1-at)^2}{c^{2x}}(\frac{e}{m})E$;$F_r=\frac{c^{2x}}{g(1-at)^2}$;$\gamma=\frac{\rho_s}{\rho},v=\frac{\mu}{\rho}$

Obviously the equations (2.1) and (2.5) reduced to following ordinary differential equations

$$H'(\eta)=-(H(\eta)F(\eta)+H(\eta)G'(\eta))/(A\frac{\eta}{2}+G(\eta)) \tag{3.1}$$

$$f'''=-ff''+(f')^2+A(f'+\frac{\eta}{2}f'')-\frac{\beta H}{(1-\varphi)}(F-f')-\frac{HM}{(1-\varphi)} \tag{3.2}$$

$$F''=[A\{\frac{\eta}{2}F'\ +F\}+(F)^2+GF'+\beta(F-f')-(1-\frac{1}{\gamma})\frac{1}{F_r}-M]\varepsilon \tag{3.3}$$

$$G''=[\frac{A}{2}(\eta G'+G)+GG'+\beta(f+G)]/\varepsilon \tag{3.4}$$

With boundary conditions

$G'=0,f=0,f'=1$, $F^i=0$ $as\ \eta\rightarrow 0$,$f'=0,F=0,G=-f,H=\omega as\eta\rightarrow\infty$

4 Heat transfer analysis:

The governing temperature equations are;

$$(1-\varphi)\rho c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=(1-\varphi)k\frac{\partial^2 T}{\partial y^2}+\frac{1}{\tau_T}\varphi\rho_s c_s(T_p-T)+\frac{1}{\tau_p}\varphi\rho_s(u-u_p)^2$$
$$+(1-\varphi)\mu(\frac{\partial u}{\partial\gamma})^2+\varphi\rho_s(\frac{e}{m})Eu_p-(1-\varphi)\frac{\partial q_r}{\partial\gamma}+(1-\varphi)q''' \tag{4.1}$$

$$\varphi\rho_s c_s\Big[\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\Big]=\frac{\partial}{\partial y}(\varphi k_s\frac{\partial T_p}{\partial y})-\frac{1}{\tau_p}\varphi\rho_s c_s(T_p-T)-\frac{1}{\tau_p}\varphi\rho_s(u-u_p)^2$$
$$+\varphi\mu_s[u_p\frac{\partial^2 u_p}{\partial y^2}+(\frac{\partial u_p}{\partial y})^2]+\varphi\rho_s(\frac{e}{m})Eu_p-\varphi\frac{\partial q_{rp}}{\partial y}+\varphi q_p''' \qquad (4.2)$$

In order to solve the temperature equations (4.1) and (4.2), we consider the non -dimensional temperature boundary conditions as follows;

$$T=T_w \ =T_\infty+T_0\frac{cx^2}{\nu(1-at)^2}\ at\ y=0$$

$$T\to T_\infty, T_p\to T_\infty as y\to\infty$$

For most of the gases $\tau_p\approx\tau_T$, $k_s=k\frac{c_s}{c_p}\frac{\mu_s}{\mu}\text{if}\frac{c_s}{c_p}=\frac{2}{3P_r}$

Rosseland approximation is assumed to account for radiate heat flux in presence of electrification of particles in the flow with internal heat generation/absorption. Using Rosseland approximation, the radiate e heat flux q_r is modeled as:

$$q_r\ =-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$$

where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient. Assuming that the differences in the temperature within the flow to be sufficiently small so that T^4 can be expressed as linear function of the temperature, one can expand T^4 in a Taylor's series about T_∞ as follows

$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\cdots$

By neglecting higher order terms beyond the first degree in $(T-T_\infty)$ we get:

$T^4=-3T_\infty^4+4T_\infty^3 T$

Substituting this value inq_r , we get

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}$$

Similarly for particle phase, we can write

$$\frac{\partial q_{rp}}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T_p}{\partial y^2}$$

Introducing the following non dimensional variables in equation (4.1) to (4.2)

$$Ra=\frac{16T_\infty^3\sigma^*}{3kk^*},q'''=(\frac{kU_w(x)}{xv})[A^*(T_w-T_\infty)f'(\eta)+B^*(T-T_\infty)]$$

$$q_p'''=(\frac{kU_w(x)}{xv})[A^*(T_w-T_\infty)F(\eta)+B^*(T_p-T_\infty)];\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty};\theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}$$

Where, $T-T_\infty=T_0\frac{cx^2}{v}\frac{1}{(1-at)^2}\theta$, $T_p-T_\infty=T_0\frac{cx^2}{v}\frac{1}{(1-at)^2}\theta_p$.

The temperature equations are reduced into the following forms:

$$\theta''=\frac{\left\{\begin{array}{l} Pr(2f'\theta-f\theta')\frac{2}{3}\frac{\beta H}{1-\varphi}(\theta_p-\theta)-\frac{PrE_c\beta H}{1-\varphi}(F-f')^2-PrE_c((f'')^2)Pr(2f'\theta-f\theta')-{\prime\prime}2 \\ +\frac{A}{2}Pr(\eta\theta'(\eta)+4\theta(\eta))-\frac{P_rE_cHMF(\eta)}{(1-\varphi)}-(A^*f'(\eta)+B^*\theta(\eta)) \end{array}\right\}}{(1+R_a)} \tag{4.3}$$

$$\theta_p''=\frac{\left\{\begin{array}{lll} \frac{A}{2}(\eta\theta_p'+4\theta_p)+2F\theta_p+G\theta_p' & +\beta(\theta_p-\theta)+\frac{3}{2}E_cP_r\beta(f'-F)^2 & \\ -\frac{3}{2}\varepsilon E_cP_r(FF'' & +(F')^2)-\frac{3}{2}E_cP_rMF- & \frac{3}{2}\frac{1}{\gamma}(A_p^*F(\eta)+B_p^*\theta_p(\eta)) \end{array}\right\}}{(\frac{\varepsilon}{P_r}+\frac{3}{2}\frac{1}{\gamma}R_a)} \tag{4.4}$$

(4.4)

With boundary condition

$\theta=1,\theta_p'=0as\eta\rightarrow 0$ and $\theta\rightarrow 0,\theta_p\rightarrow 0as\eta\rightarrow\infty$

5 Solution Method:

Here the set of non-linear differential Equations (3.1) to (3.4), (4.3) and (4.4) subject to the boundary conditions were solved numerically using Runge-Kutta method with shooting technique. In this problem the values of $f''(0)$, $F(0)$, $G(0)$, $H(0)$, $\theta'(0)$, $\theta_p(0)$ are not known but $f'(\infty)=0,F(\infty)=0,G(\infty)=-f(\infty)$, $H(\infty)=\omega,\theta(\infty)=0,\theta_p(\infty)=0$ are given. Shooting method is used to determine the values of $f''(0)$, $F(0)$, $G(0)$, $H(0)$, $\theta'(0)$, $\theta_p(0)$ .In this problem the missing values of $f''(0)$, $\theta'(0)$, $F(0)$, $G(0)$, $H(0)$ and $\theta_\mathrm{p}(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity $(\eta_\infty)$are satisfied. In order to determine $\eta \to \infty$for the boundary value problem described by Equations (3.1) to (3.4), (4.3) and (4.4).For instance, here values of $f''(0) = \infty 0$and$f''(0) = \infty 1$ have supplied .The improved value of $f''(0) = \infty 2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\alpha_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\alpha_2, \infty)$ is equal to$f'(\infty)$ up to a certain decimal accuracy, then α_2 i.e $f''(0)$ is determined, otherwise the above procedure is continued with $\alpha_0 = \alpha_1$ *and* $\alpha_1 = \alpha_2$ until a correct α_2 is obtained. The same procedure described above is adopted to determine the correct values ofF (0), $G(0)$, $H(0)$, $\theta'(0)$, $\theta_p(0)$. Depending upon the initial guess and number of steps N = 81.0, the solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $F(0)$, $G(0)$, $H(0)$, $\theta_p(0)$, $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as $\eta = 5.0$ with a step length of $\Delta\eta = 0.125$ beginning from $\eta = 0$ to ensure to be the satisfactory convergence criterion of 1×10^{-6} to achieve the far field boundary conditions asymptotically for all values of the parameters considered. For the sake of brevity further details on the solution process are not presented here. The accuracy of the numerical scheme has been validated by comparing the skin friction and the wall temperature gradient results to those reported in the previous studies. The following table shows an excellent agreement between our numerical results and the previously reported results. The accuracy of results obtained by our investigation is compared with the results obtained by C.H.Chen[10],L.J.Grubka et.al.[19], A.Subhas et.al.[4],S.Mukhopadhaya et.al.[34], A.Ishak et. a1.[3] and B.J.Gireesha et. a1.[8] for the local nusselt number in the limiting condition $\beta, A^*, B^*, A_p^*, B_p^*, R_a, G_r, E_c, A, M, \phi = 0.0$. It is observed from the table that our result is good agreement with their results.

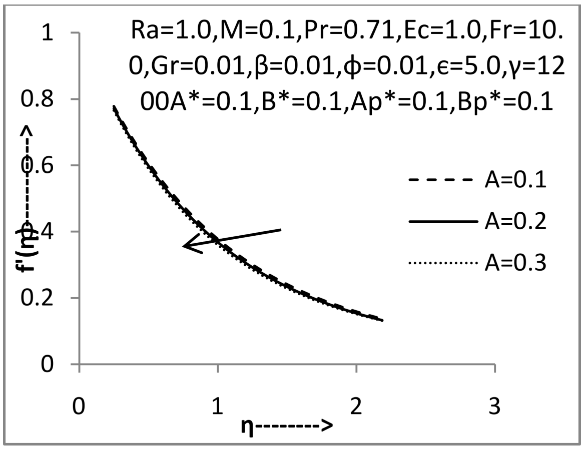

Figure 1: Non-dimensional velocity profile of fluid phase w.r.t A

Table 1: Comparison of skin friction coefficient $-f''(0)$ for $\beta, A^*, B^*, A_p^*, B_p^*, R_a, G_r, E_c, A, M, \phi = 0.0$

P_r	B.J. Gireesha et.a1.[8]	T.Hayat [38]	S.O. Salwa [36]	Rahaman [23]	Present study -f''(0)
0.72	1.002469	1.000000	1.001986	1.002125	1.001396

6 Discussion of Result:

The computations were done by the computer language FORTRAN -77. The shear stress(Skin friction coefficient) which is proportional to $f^{ii}(0)$ and rate of heat transfer(Nusselt number) which is proportional to $\theta^{i}(0)$ are tabulated in Table -3 respectively for different values of parameters used.It is observed from the table that shear stress and rate of heat transfer decrease on the increase of Ec' .The shear stress and rate of heat transfer increase for increasing values of Pr. The Nusselt number increases on the increasing values of unsteady parameter 'A' and electrification parameter 'M', but Nusselt number decreases when the radiation parameter 'Ra' increases. The skin friction coefficient decreases for increasing value of M$'$.

Table 2: Showing initial values of wall velocity gradient $-f''(0)$ and temperature $gradient - \theta'(0)$

A	A*	B*	A_p^*	B_p^*	M	E_c	P_r	Ra	$-f''(0)$	$-\theta'(0)$
0	0	0	0	0	0	0	0.72	0	1.0013296	1.082315
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
0.2									1.048937	0.597137
0.3									1.084461	0.640866
	-0.1								1.012232	0.619318
	0.0								1.012135	0.585140
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
		-0.1							1.012409	0.641113
		0.0							1.012396	0.595400
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
			-0.1						1.011785	0.553445
			0.0						1.011789	0.553446
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
				-0.1					1.011788	0.553438
				0.0					1.011789	0.553443
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
					0.0				1.029781	0.536832
					0.05				1.020120	0.544887
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
						2.0			1.011686	0.414141
						3.0			1.011387	0.277564
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.71	1.0	1.011790	0.553447
							1.0		1.012418	0.689271
							2.0		1.013572	1.041177
0.1	0.1	0.1	0.1	0.1	0.1	1.0	0.71	1.0	1.011790	0.553447
								2.0	1.011573	0.449928
								3.0	1.011086	0.397243

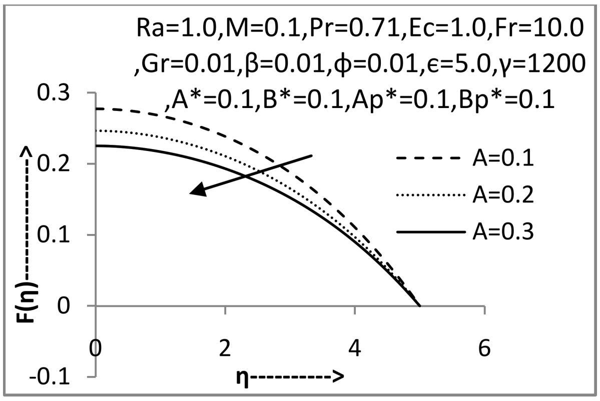

Figure 2: Non-dimensional velocity profile of particle phase w.r.t A

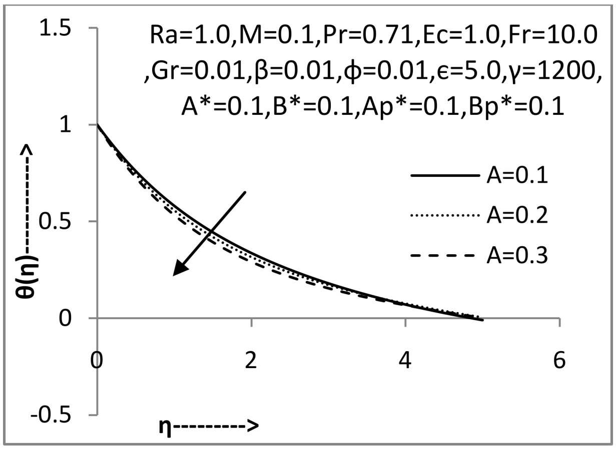

Figure 3: Non-dimensional temperature profile of fluid phase w.r.t A

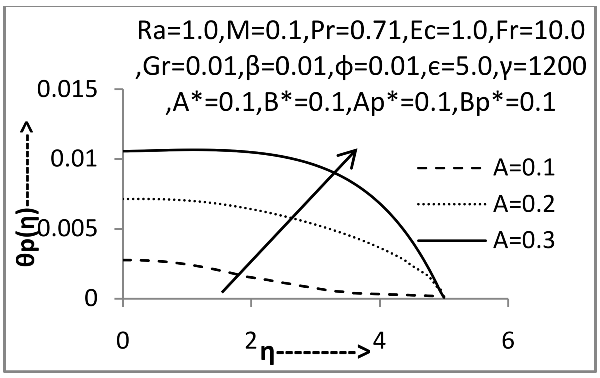

Figure 4: Non-dimensional temperature profile of particle phase w.r.t A

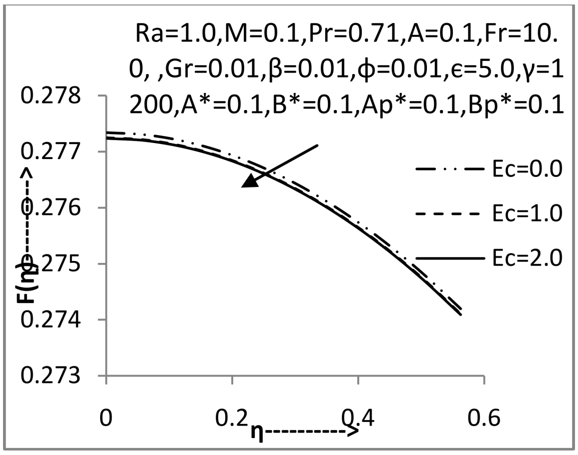

Figure 5: Non-dimensional velocity profile of particle phase w.r.t 'Ec'

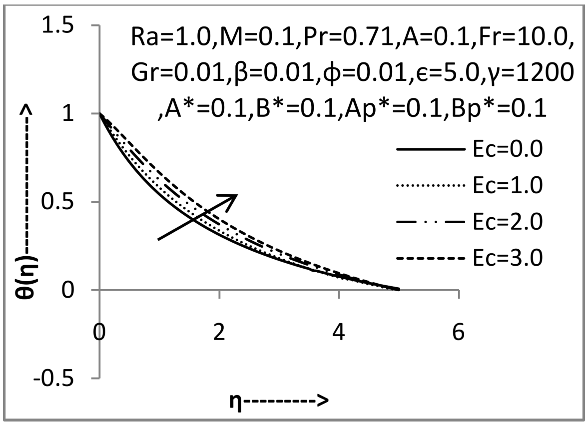

Figure 6: Non-dimensional temperature profile of fluid phase w.r.t 'Ec'

Fig-l and Fig-2 demonstrate the velocity profile of fluid phase and particle phase respectively. It is observed that the velocity of both phases decrease on the increase of unsteady parameter 'A'. This is because of fluid flow caused by stretching sheet. Fig-3 explains that the temperature of fluid phase decreases with increasing of unsteady parameter 'A'. Physically, when the unsteadiness increases the sheet loses more heat which causes decrease in temperature. We also observe that as the distance from the stretching sheet within dynamic region increases, temperature field decreases as unsteadiness increases. But Fig −4 describes that the temperature profile of particle phase increases on the increase of unsteady parameter because getting more time of collision of particles produce more frictional heating.

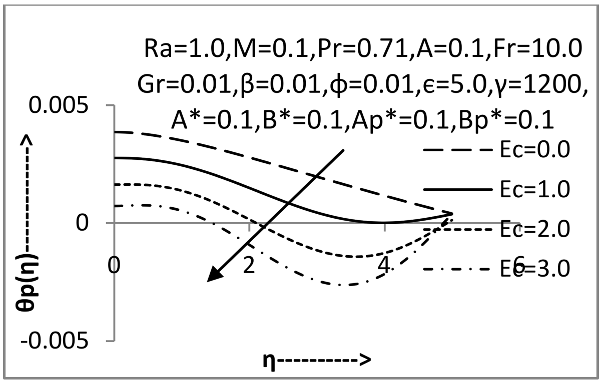

Figure 7: Non-dimensional temperature profile of particle phase w.r.t 'Ec'

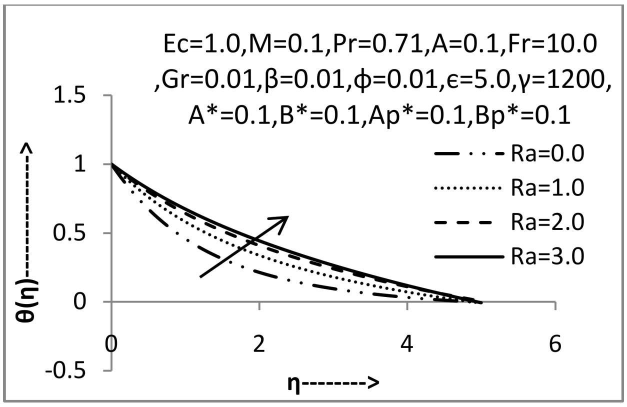

Figure 8: Non-dimensional temperature profile of fluid phase w.r.t. 'Ra'

Fig-5 depicts the velocity profile of particle phase which shows that the increasing value of Ec decreases the velocity profile of particle phase. Fig-6 explain that the increasing values of Ec, increases the temperature of fluid phase phase because the large values of Ec give rise to a strong viscous dissipation effect as a result the heat energy is stored in the fluid which enhances the temperature and thermal boundary layer thiclmess, but the Fig-7 shows that increasing values of Ec, decrease the temperature of particle phase.

Fig-8 and Fig-9 depict the variation of temperature of fluid phase and particle phase respectively. It is observed that the enhancing value of Radiation parameter 'Ra' enhances significantly the temperature of both fluid phase and particle phase. This is due to the fact that the divergences of the radiate heat flux

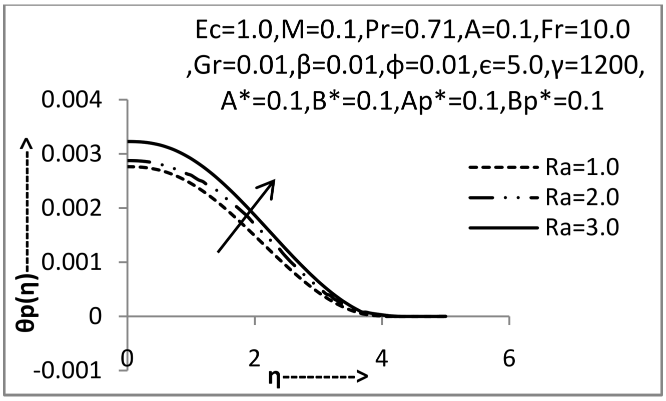

Figure 9: Non-dimensional temperature profile of particle phase w.r.t. 'Ra'

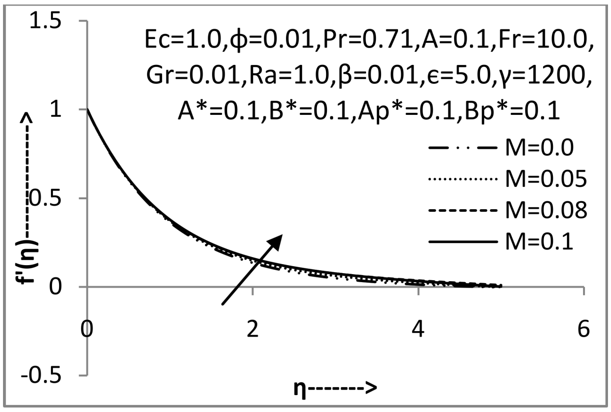

Figure 10: Non-dimensional velocity profile of fluid phase w.r.t. 'M'

increases as the Rosseland radiate absorptionk ∗diminishes which in turn increases the rate of radiate heat transfer to the fluid. Thus the presence of thermal radiation enhances thermal state of the fluid causing its temperature to increase. It means the thermal boundary layer thickness is increasing on the increase of radiation parameter.

Fig-10 and Fig-11 represent the increase of Electrification parameter 'M' increase the velocity of both fluid phase and particle phase respectively. Fig-12 demonstrates the temperature of fluid phase. It is noticed that the temperature of fluid phase decrease with increase of Electrification parameter 'M'. But Fig-13 reveals that the temperature of particle phases increase on the increase of electrification parameter 'M'. The enhance value of M is enhancing the temperature of particle phase due to the applied transverse

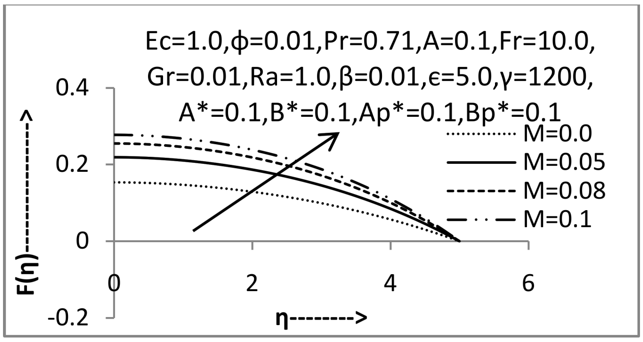

Figure 11: Non-dimensional velocity profile of particle phase w.r.t. ‘M’

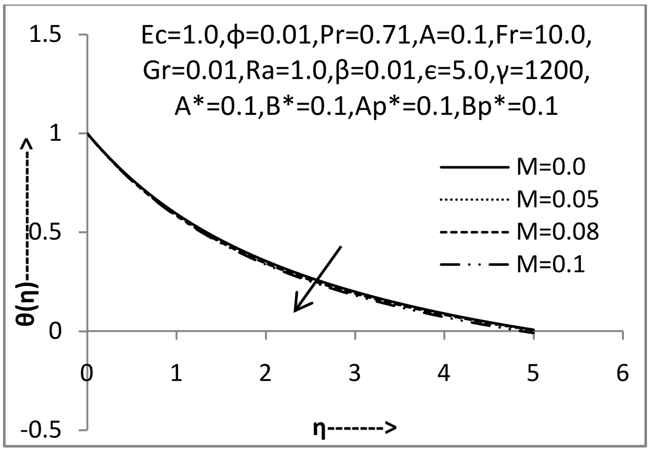

Figure 12: Non-dimensional temperature profile of fluid phase w.r.t. ‘M’

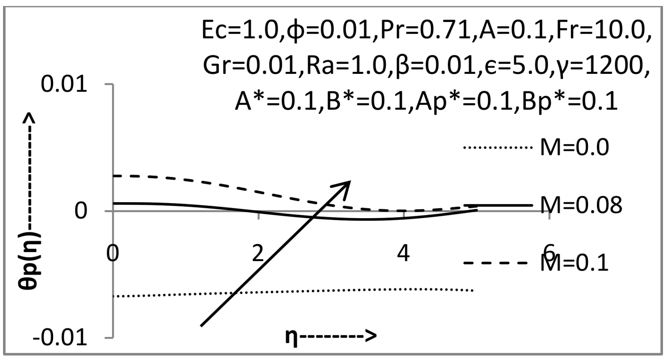

Figure 13: Non-dimensional temperature profile of particle phase w.r.t. ‘M’

Figure 14: Non-dimensional velocity profile of particle phase w.r.t. 'Pr'

Figure 15: Non-dimensional temperature profile of fluid phase w.r.t. 'Pr'

electric field opposing the motion.

Fig-14 indicates the increase of velocity profile particle phase with increase of Pr .Fig-15 and Fig-16 depicts the effect of Pr on temperature profile of fluid phase and particle phase. From the figure we observe that, when Pr increases the temperature of both phases decreases .This is due to the fact that for smaller values of Pr are equivalent to larger values of thermal conductivities, this phenomenon leads to the decreasing of energy ability that reduces the thermal boundary layer and therefore heat is able to diffuse away from the stretching sheet.

Fig-17 and Fig-18 demonstrates that the temperature of both phases increase with increasing value of A^*.Because the presence of the heat source generates energy in the thermal boundary layer and as a consequence the temperature rises. In the case of heat absorption $A^* < 0$ the temperature falls with decreasing values of$A^* < 0$ owing to the absorption of energy in the boundary layer.

Fig-19 and Fig-20 explains the behavior of temperature of both phases w.r.t. $B*$ which indicates that the increase value of B^* increase the temperature of both phases. As in the case of space dependent heat source the temperature increases due to the release of thermal energy for $B^* > 0$ while the temperature drops for decreasing values of $B^* < 0$ owing to the absorption of energy.

Fig-21 demonstrates that the temperature of fluid phases increase with increasing value of Ap^*.Because the presence of the heat source generates energy in the thermal boundary layer and as a consequence the temperature rises. In the case of heat absorption $Ap^* < 0$ the temperature falls with decreasing values of $Ap^* < 0$ owing to the absorption of energy in the boundary layer. Fig-22 demonstrates that the tem-

Figure 16: Non-dimensional temperature profile of particle phase w.r.t. 'Pr'

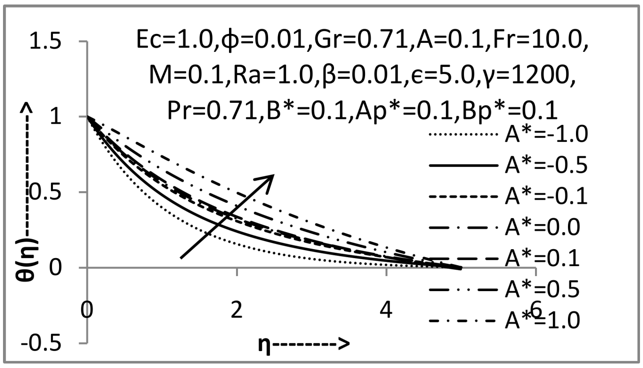

Figure 17: Non-dimensional temperature profile of fluid phase w.r.t. A*

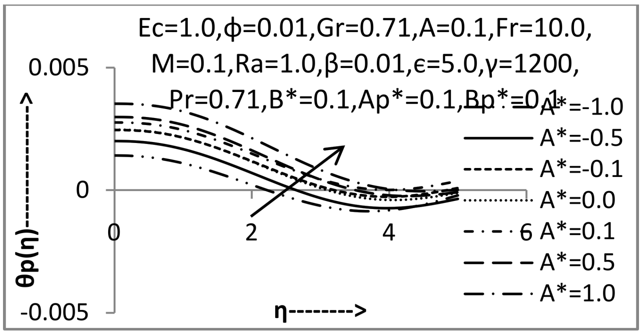

Figure 18: Non-dimensional temperature profile of particle phase w.r.t. A*

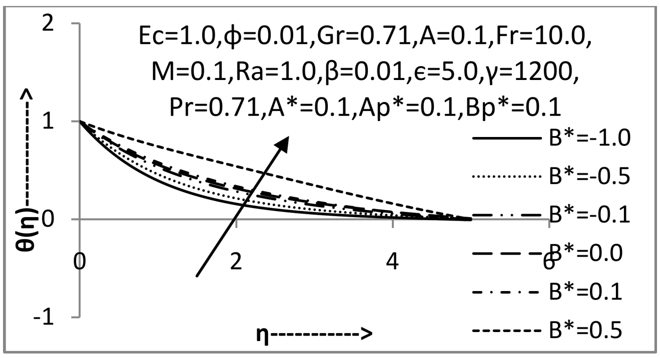

Figure 19: Non-dimensional temperature profile of fluid phase w.r.t. B*

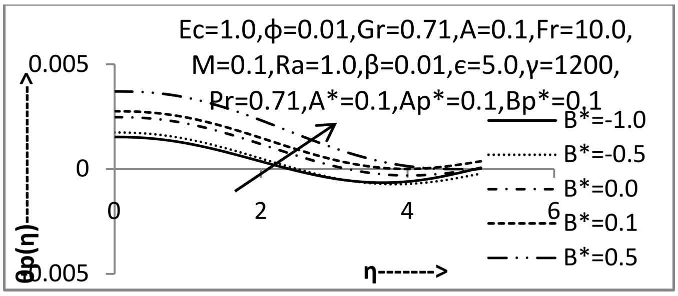

Figure 20: Non-dimensional temperature profile of particle phase w.r.t. B^*

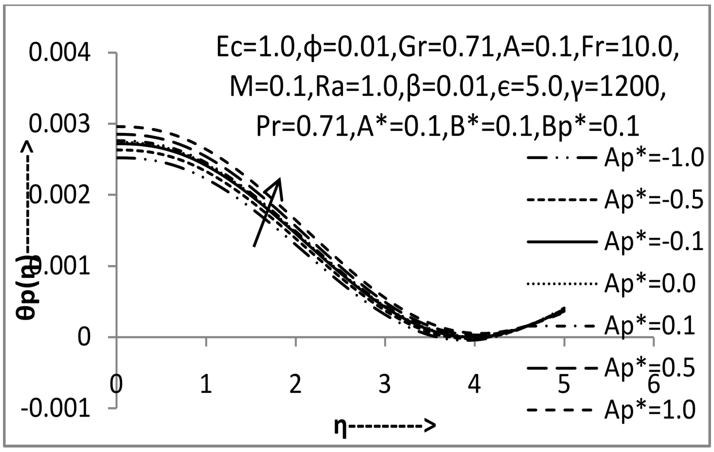

Figure 21: Non-dimensional temperature profile of particle phase w.r.t. Ap^*

Figure 22: Non-dimensional temperature profile of particle phase w.r.t. Bp^*

perature of particle phases increase with increasing value of Bp^*.As in the case of space dependent heat source the temperature increases due to the release of thermal energy for $Bp^* > 0$ while the temperature drops for decreasing values of $Bp^* < 0$ owing to the absorption of energy.

7 Conclusions:

Therefore, it is concluded that the boundary layer flow and heat transfer characteristics have been discussed extensively by considering various physical parameters like unsteady parameter 'A', Prandtl number ' Pr',Eckert number 'Ec', Electrification parameter 'M', Radiation parameter 'Ra', heat source/sink parameters A^* and B^*, volume fraction φ' and particle interaction parameter β', ratio of specific heat γ ,diffusion parameter 's', Froud number $\Gamma r'$. The novelty of the study is the consideration of electrification of particles and diffusion parameter that are scarcely investigated by other researchers. Our model based on the motion and collisions of particles, as well as the charge transfer during these collisions. Here the focus is made on the role of the inter particle electrostatic forces which has given less attention by other investigators. Though the computations are cumbersome due to the consideration of many parameters, but consideration of all these parameters may give a new dimension to the study of stretching sheet problems. These areas have potential

applications in industries which have important contributions for the progress of the society. Another important investigation of the present study is the inclusion of radiation term and heat source/sink term in particle phase .The influence of these terms are same as that of fluid phase which has interpreted in detail. In our study the claim is that the velocity profile increases on the increase of electrification parameter which arises due to electrification of particles, whereas, the analysis made by authors taking electrically conducting fluid in presence of external magnetic field shows a decrease in fluid velocity due to presence of opposing Lorentz force .The increasing value of unsteady parameter A decreases the temperature profiles of fluid phase and increase the temperature of dust phase. Also increase value of A decreases velocity of both phases. The rate of cooling is much faster for higher values of unsteady parameter but it takes long times for cooling during the steady flow.

Finally, we observed the following results:

1. The increasing value of unsteady parameter A decreases the temperature profiles of fluid phase and increase the temperature of dust phase. Also increase value of A decreases velocity of both phases. The rate of cooling is much faster for higher values ofunsteady parameter but it takes long times for cooling during the steady flow.

2. The temperature profile of both phases increase with the increase of radiation parameter Ra. Thus the radiation should be at minimum in order to facilitate the cooling process.

3. Increasing value of Ec is enhancing the temperature of both fluid phase but decrease the temperature of particle phase which indicates that the heat energy is generated in fluid due to frictional heating.

4. The thermal boundary layer thickness decreases on the effect of Pr. The temperature decreases at a faster rate for higher values of Pr which implies the rate of cooling is faster in case of higher prandtl number.

5. The thermal boundary layer thickness for fluid phase and momentum boundary layer thickness for both phases are increasing for increased values of electrification parameter M.

6. The problem has investigated by assuming the values $\gamma = 1200.0, F_r = 10.0$

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